Periodical Structures: Bloch Waves, Band-Gap Spectrum, Bloch Oscillations, Landau-Zener Tunneling between Bloch Bands…

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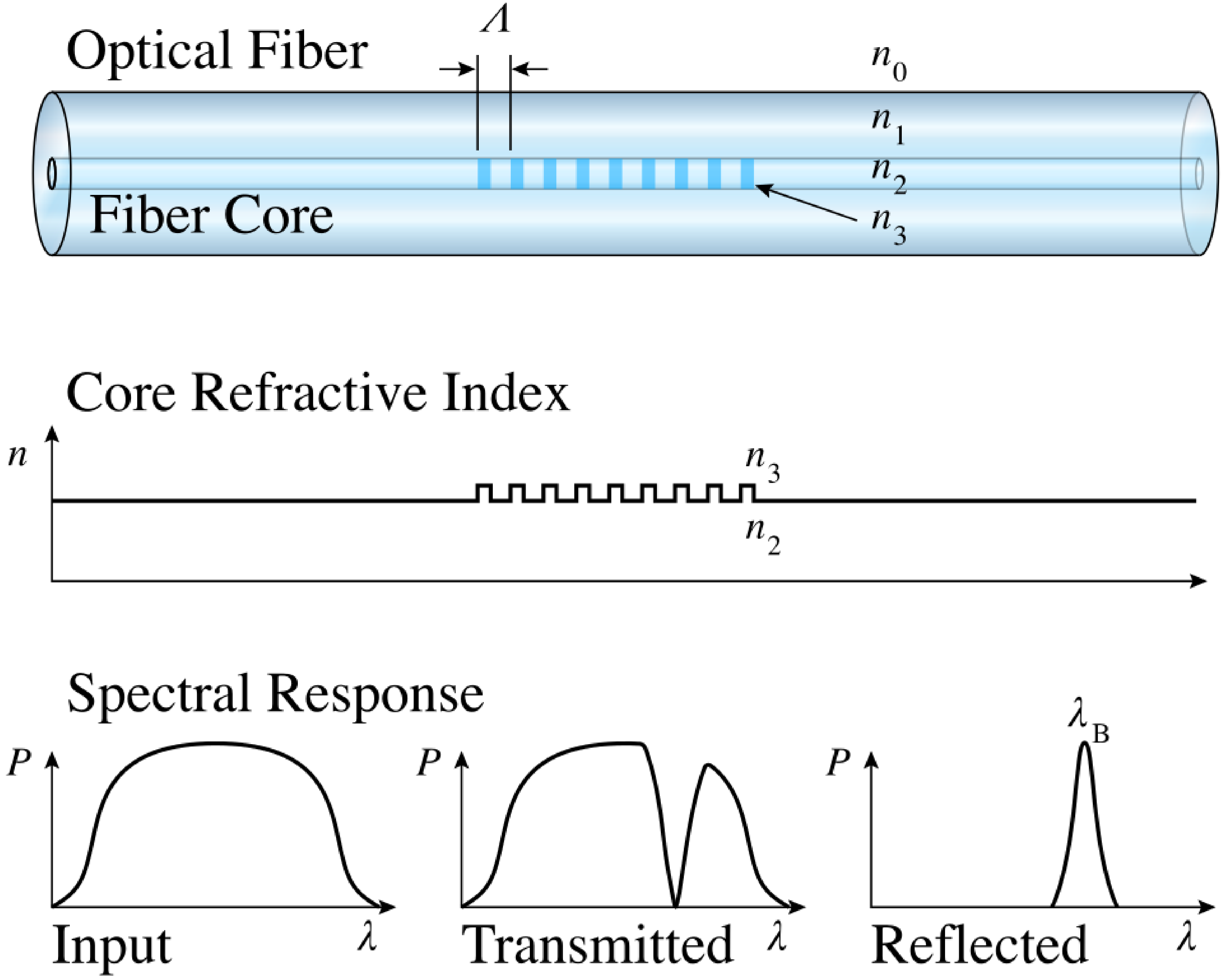
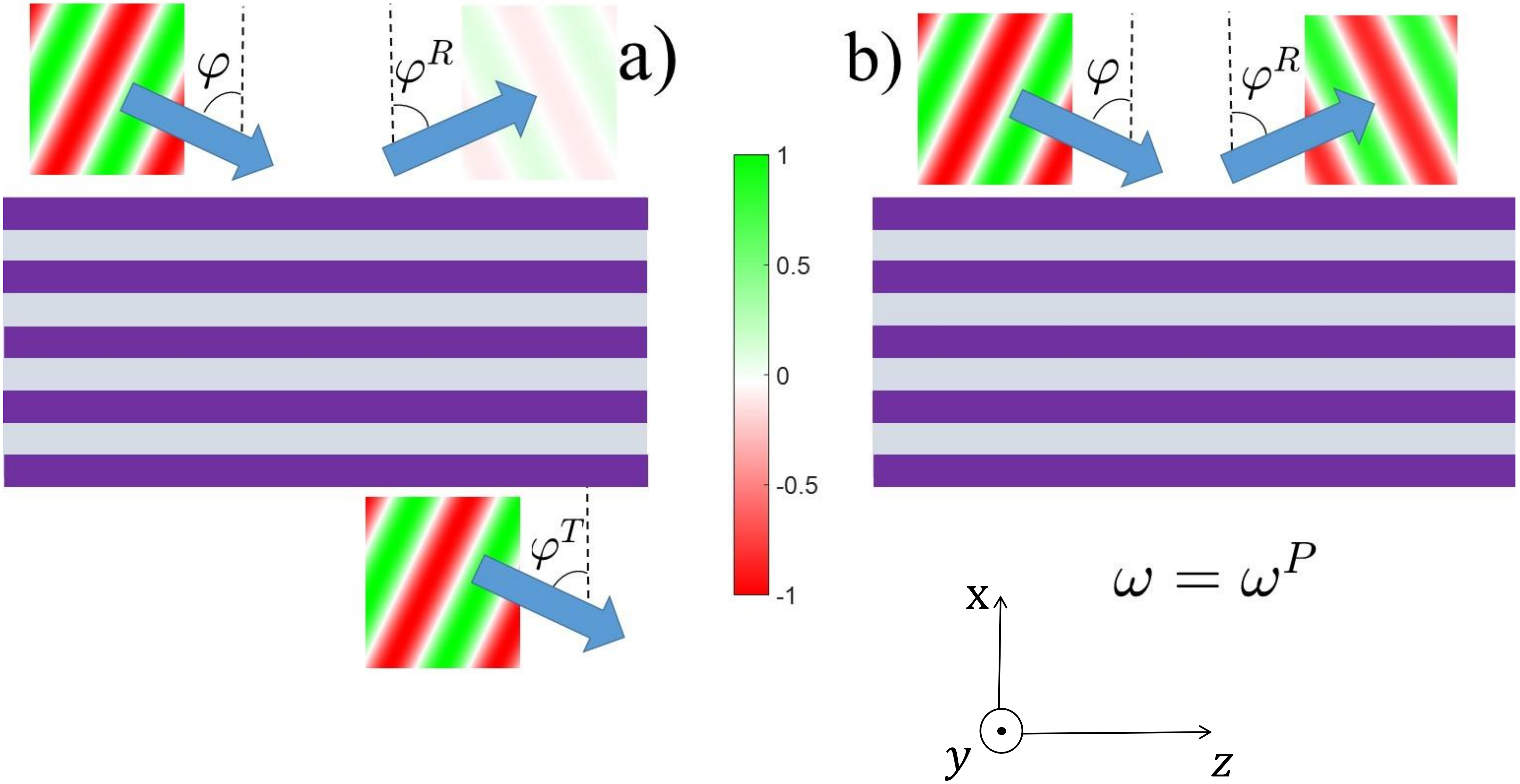
Bragg Gratings – Refraction Index is Periodic. Periodicity Wavenumber $Q = 2\pi/\Lambda$

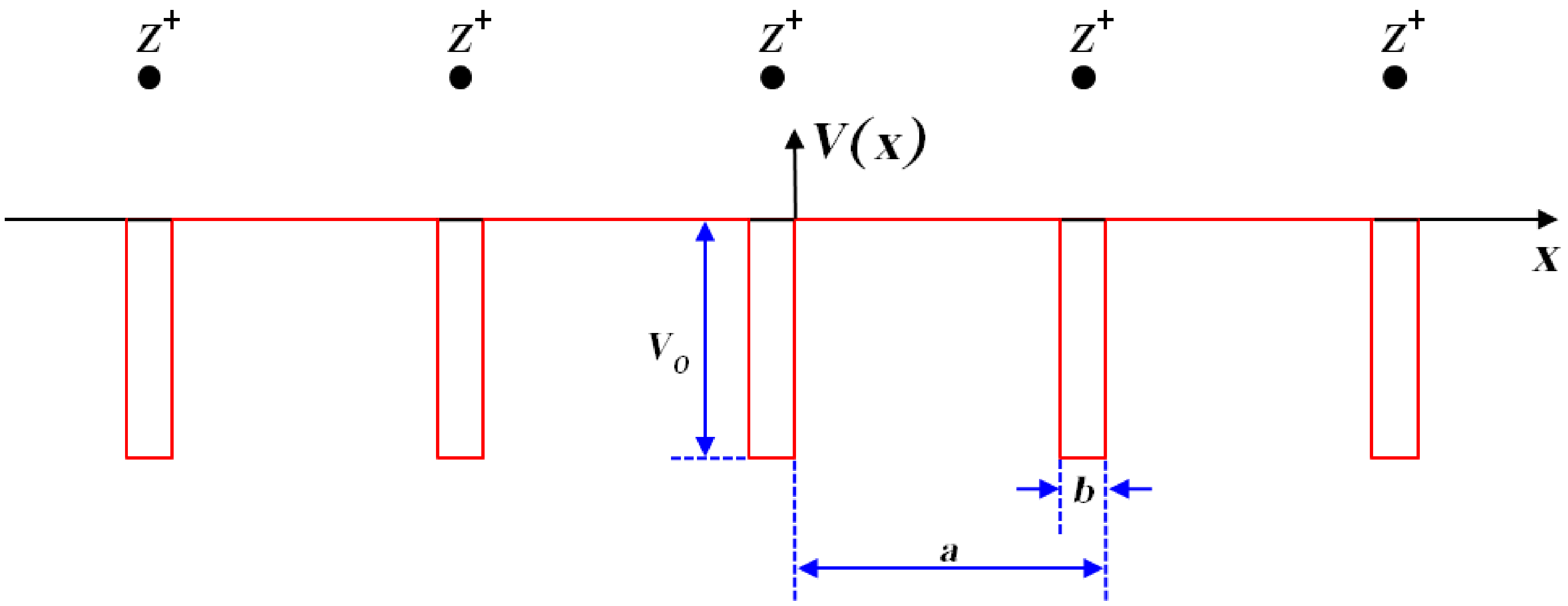


Two Dimensional Problem



Particle in a Periodic Potential – Periodicity wavenumber $Q = 2\pi/a$

Schrödinger equation - $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi$ Kronig–Penney model



Schrödinger equation - $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi$

$$V(x) = \sum_{s=-\infty}^{\infty} V_n e^{iQsx}; \qquad Q = \frac{2\pi}{a}$$

$$V(x) = 0 \quad \Rightarrow \quad i\frac{\partial\Psi}{\partial t} = -\frac{\partial^2\Psi}{\partial x^2} \quad \Rightarrow \quad \Psi = Ae^{i(kx-\omega t)} \quad \Rightarrow \quad \omega = k^2$$

Bloch Theorem: $\Psi = u(x)e^{i(kx-\omega t)}$ with $u(x)$ Periodic

$$i\frac{\partial\Psi}{\partial t} = -\frac{\partial^2\Psi}{\partial x^2} + V_1\left(e^{iQx} + e^{-iQx}\right)\Psi$$

$$\Psi = Ae^{i(kx-\omega t)}$$

$$\Psi = e^{i(kx-\omega t)} \sum_{s=-\infty}^{\infty} A_s e^{-iQsx}$$

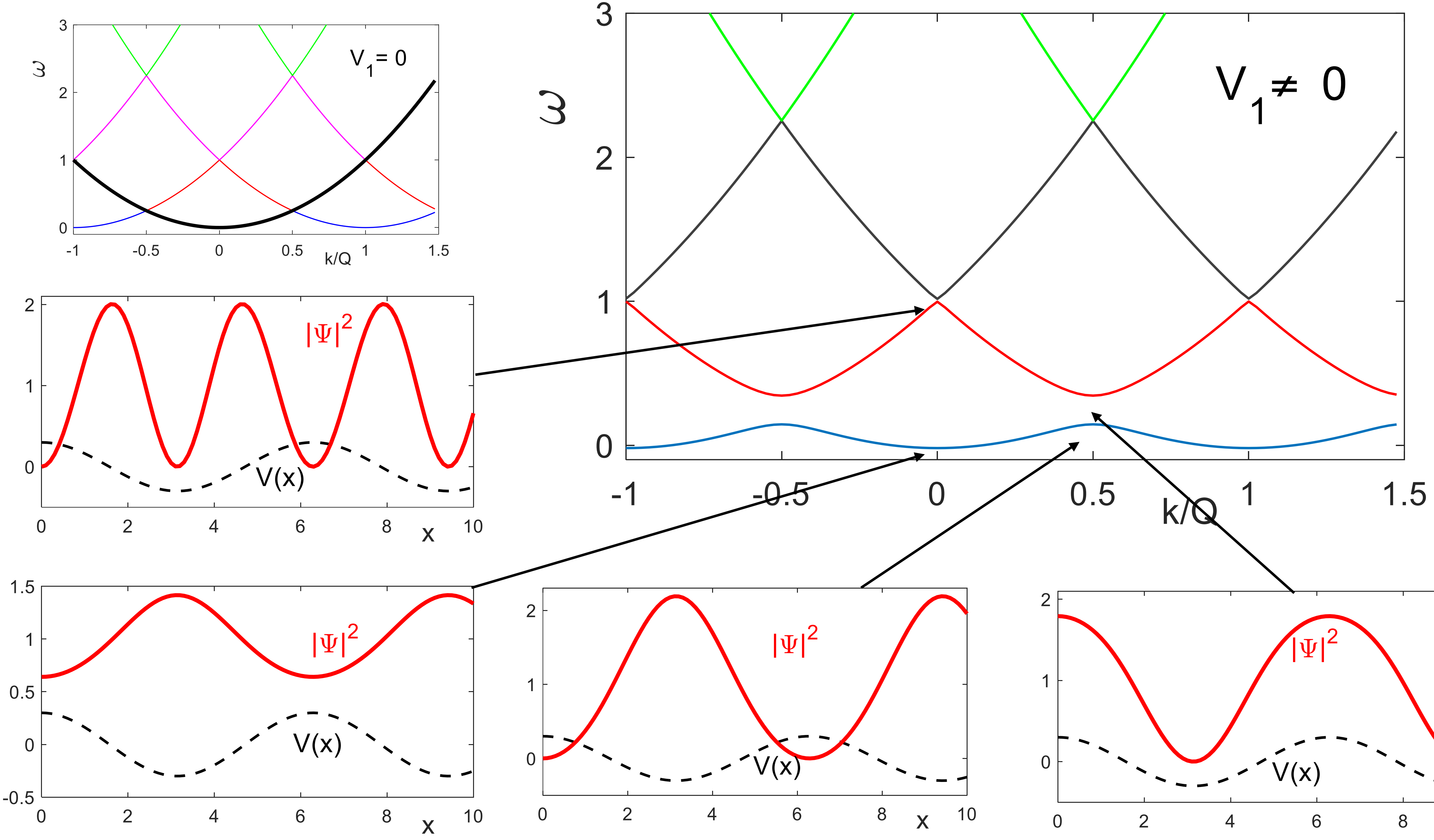
$$e^{i(kx-\omega t)}\left\{\sum_{s=-\infty}^{\infty} A_s([k-sQ]^2-\omega)e^{iQsx} + V_1\sum_{s=-\infty}^{\infty}(A_{s+1}+A_{s-1})e^{iQsx}\right\} = 0$$

$$A_s([k-sQ]^2-\omega) + V_1(A_{s+1}+A_{s-1}) = 0$$

$$
Det\begin{pmatrix}
\ddots & & & \vdots & & & \\
 & [k+2Q]^2-\omega & V_1 & 0 & 0 & 0 & \\
 & V_1 & [k+Q]^2-\omega & V_1 & 0 & 0 & \\
\cdots & 0 & V_1 & k^2-\omega & V_1 & 0 & \cdots \\
 & 0 & 0 & V_1 & [k-Q]^2-\omega & V_1 & \\
 & 0 & 0 & 0 & V_1 & [k-2Q]^2-\omega & \\
 & & & \vdots & & & \ddots
\end{pmatrix}=0
$$

$$\omega(k)=\omega(k+sQ)$$

Dispersion Relation and Wavefunctions



Approximate Solution

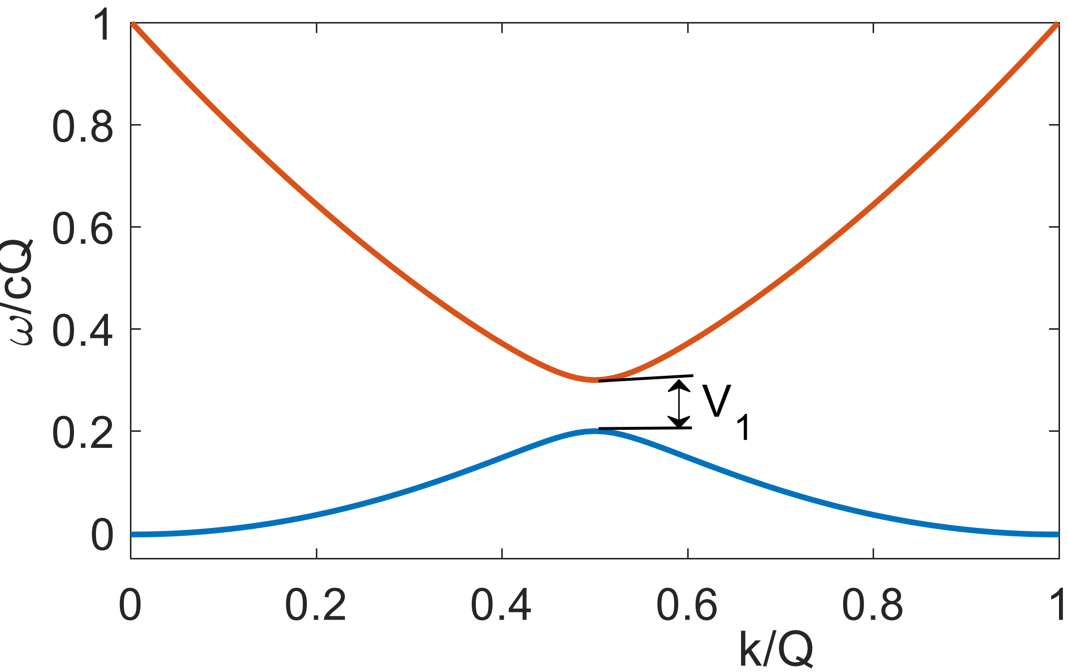
$$A_s([k - sQ]^2 - \omega) + V_1(A_{s+1} + A_{s-1}) = 0$$

$$A_1([k - Q]^2 - \omega) + V_1(A_0 + A_2) = 0$$
$$A_0(k^2 - \omega) + V_1(A_1 + A_{-1}) = 0$$

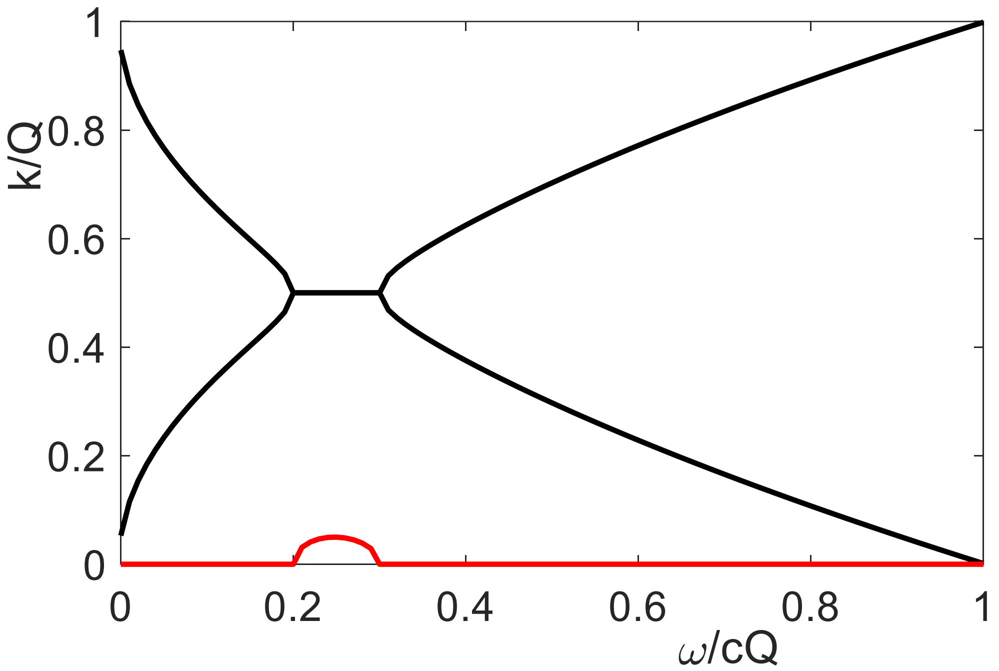
$$([k - Q]^2 - \omega)A_1 + V_1 A_0 = 0$$
$$V_1 A_1 + (k^2 - \omega)A_0 = 0$$

$$(k^2 - \omega)([k - Q]^2 - \omega) - V_1^2 = 0$$

$$\omega = \frac{k^2 + [k - Q]^2 \pm \sqrt{(k^2 - [k - Q]^2)^2 + 4V_1^2}}{2}$$



$$(k^2-\omega)([k-Q]^2-\omega)-V_1^2=0$$



$$k=\frac{Q}{2}\pm\sqrt{\omega+\frac{Q^2}{4}-\sqrt{\omega Q^2+V_1^2}}$$

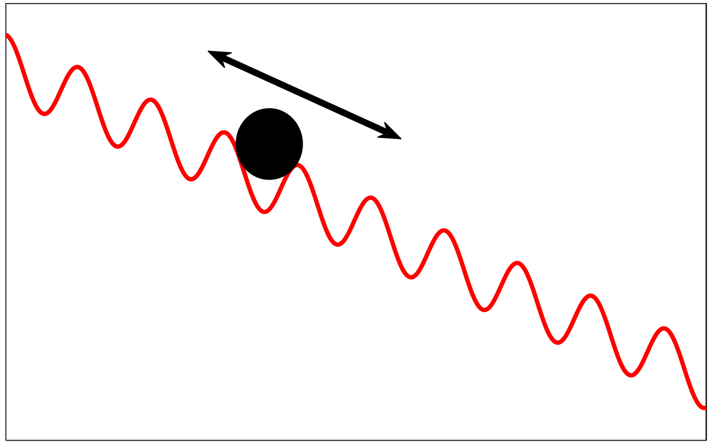
$$A_s([k-sQ]^2-\omega)+V_1(A_{s+1}+A_{s-1})=0$$

$$A_2=-(A_1+A_3)\frac{V_1}{([k-2Q]^2-\omega)}$$

$$A_2\sim\frac{V_1}{Q},\quad A_3\sim\left(\frac{V_1}{Q}\right)^2\ \ldots..$$

Bloch Oscillations

$$V(x) = bx + V_1\left(e^{iQx} + e^{-iQx}\right)$$

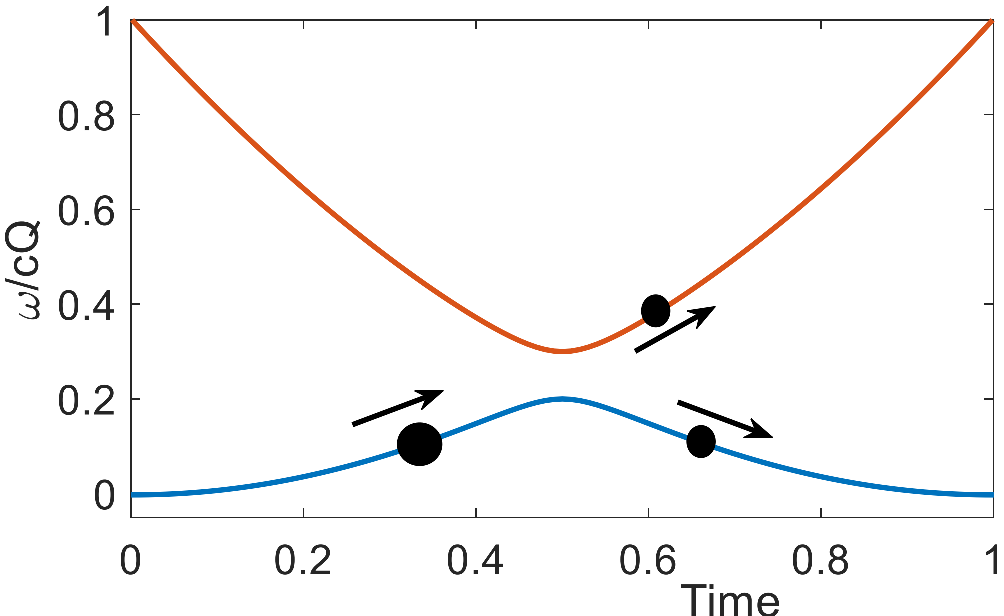


$$i\frac{\partial\Psi}{\partial t} = -\frac{\partial^2\Psi}{\partial x^2} + V_1\left(e^{iQx} + e^{-iQx}\right)\Psi + bx\Psi$$

$$\Psi \rightarrow \Psi e^{-ibxt}$$

$$i\frac{\partial\Psi}{\partial t} = -\left(\frac{\partial}{\partial x} - ibt\right)^2\Psi + V_1\left(e^{iQx} + e^{-iQx}\right)\Psi$$

$$\Psi = e^{i\left[kx - \int \omega(t)dt\right]}\sum_{s=-\infty}^{\infty} A_n e^{-iQsx}$$



$$\omega(k) \rightarrow \omega(k - bt)$$

Optical Gratings and Waveguides

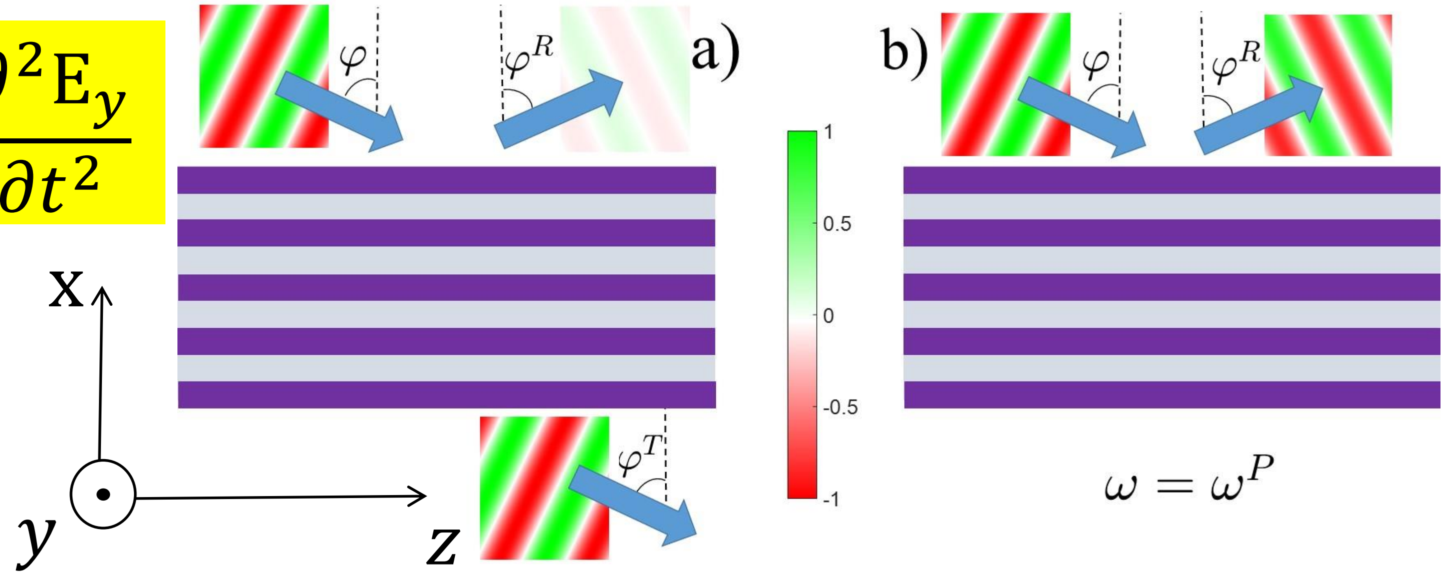
$$\nabla \times \mathbf{E} = -\mu_0 \dot{\mathbf{H}}, \qquad \nabla \times \mathbf{H} = \varepsilon_0 \dot{\mathbf{E}} + \dot{\mathbf{P}}$$

$$\Delta \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) = -\omega^2 \mu_0 \hat{\boldsymbol{\epsilon}} \mathbf{E}$$

Periodical Electrical Permittivity

$$\Delta \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) = \mu_0 \hat{\boldsymbol{\epsilon}} \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_0 \epsilon(x) \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} = \frac{n_0^2}{c^2} [1 + \delta n(x)] \frac{\partial^2 E_y}{\partial t^2}$$



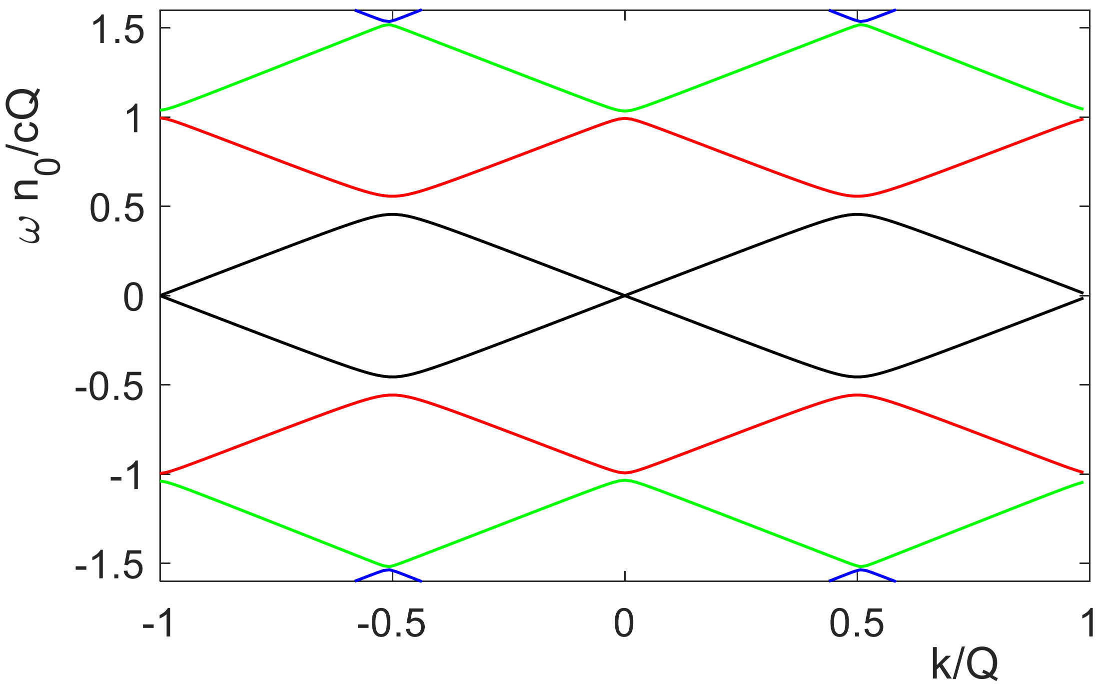
Normal Incidence

$$\frac{\partial^2 \mathrm{E}_y}{\partial x^2} + \cancel{\frac{\partial^2 \mathrm{E}_y}{\partial z^2}} = \frac{n_0^2}{c^2}\left[1 + \delta n(x)\right]\frac{\partial^2 \mathrm{E}_y}{\partial t^2}$$

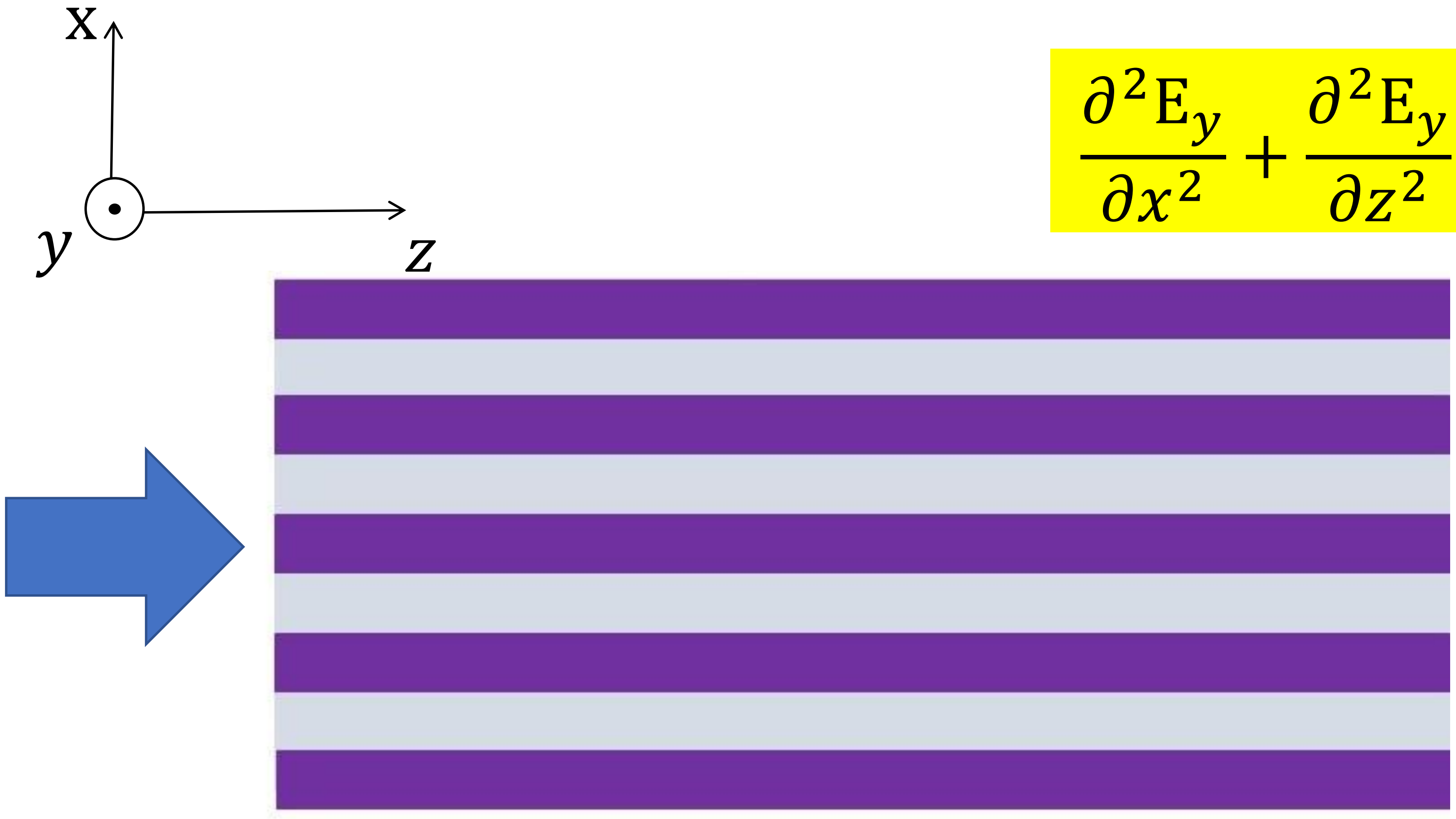
$$\delta n(x) = \delta n_0\left(e^{iQx} + e^{-iQx}\right)$$

$$E_y = e^{i(kx-\omega t)} \sum_{s=-\infty}^{\infty} A_s e^{-iQsx}$$

$$\begin{matrix}
[k+2Q]^2-\omega^2 & \delta n_0\omega^2 & 0 & 0 & 0 \\
\delta n_0\omega^2 & [k+Q]^2-\omega^2 & \delta n_0\omega^2 & 0 & 0 \\
0 & \delta n_0\omega^2 & k^2-\omega^2 & \delta n_0\omega^2 & 0 \\
0 & 0 & \delta n_0\omega^2 & [k-Q]^2-\omega^2 & \delta n_0\omega^2 \\
0 & 0 & 0 & \delta n_0\omega^2 & [k-2Q]^2-\omega^2
\end{matrix}$$



$$\frac{\partial^2 \mathrm{E}_y}{\partial x^2} + \frac{\partial^2 \mathrm{E}_y}{\partial z^2} = \frac{n_0^2}{c^2}[1 + \delta n(x)]\frac{\partial^2 \mathrm{E}_y}{\partial t^2}$$



$$\mathrm{E}_y = \Psi(z, x) e^{i(k_z z - \omega t)}$$

$$k_z = \frac{n_0}{c}\omega$$

$$2ik_z \frac{\partial \Psi}{\partial z} = -\frac{\partial^2 \Psi}{\partial x^2} + V_1\left(e^{iQx} + e^{-iQx}\right)\Psi; \qquad V_1 \equiv \delta n_0 k_z^2$$

$$i\frac{\partial \Psi}{\partial t} = -\frac{\partial^2 \Psi}{\partial x^2} + V_1\left(e^{iQx} + e^{-iQx}\right)\Psi$$

Vizualization of Quantum Effects

$$2ik_z \frac{\partial \Psi}{\partial z} = -\frac{\partial^2 \Psi}{\partial x^2} + V_1\left(e^{iQx} + e^{-iQx}\right)\Psi; \qquad V_1 \equiv \delta n_0 k_z^2$$

$$\Psi = e^{i(kx-\beta z)} \sum_{s=-\infty}^{\infty} A_s e^{-iQsx}$$

Bloch Oscillations and Landau-Zener Tunneling in Space Coordinates

$$2ik_z \frac{\partial \Psi}{\partial z} = -\frac{\partial^2 \Psi}{\partial x^2} + V_1\left(e^{iQx} + e^{-iQx}\right)\Psi - bx\Psi$$

Optical Bloch Oscillations in Temperature Tuned Waveguide Arrays

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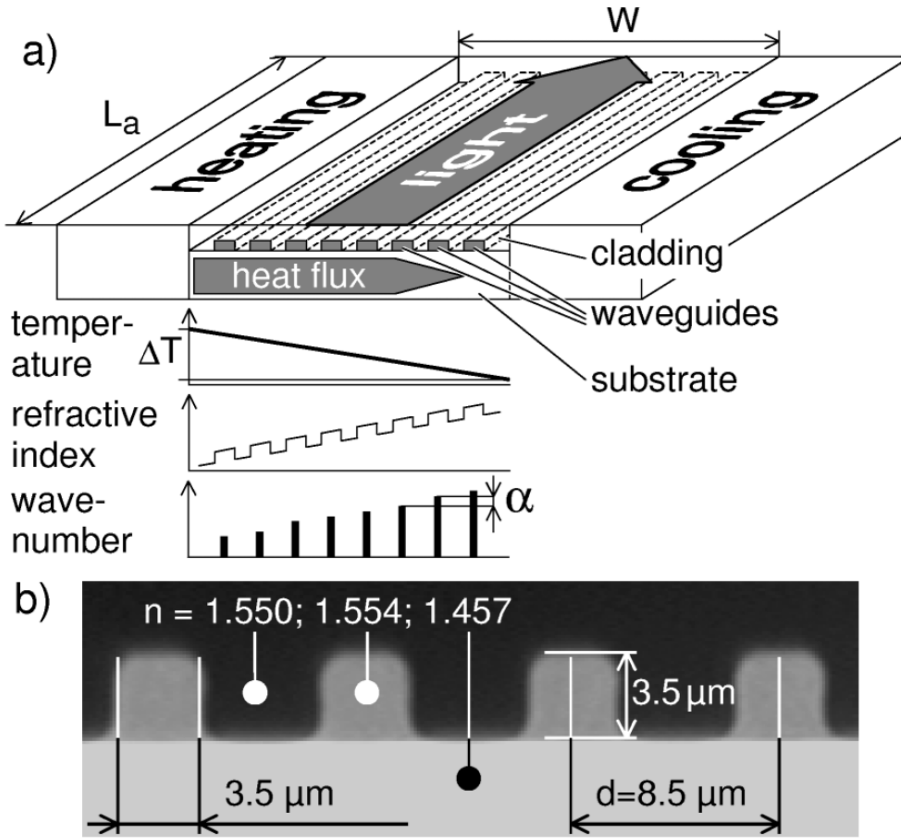

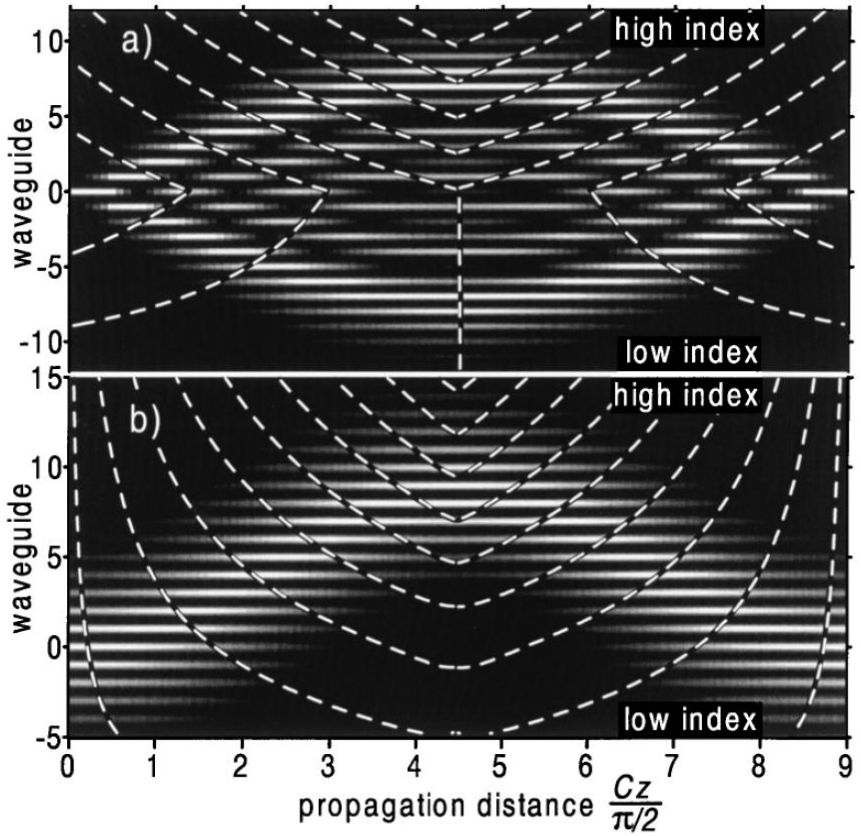

Visual Observation of Zener Tunneling

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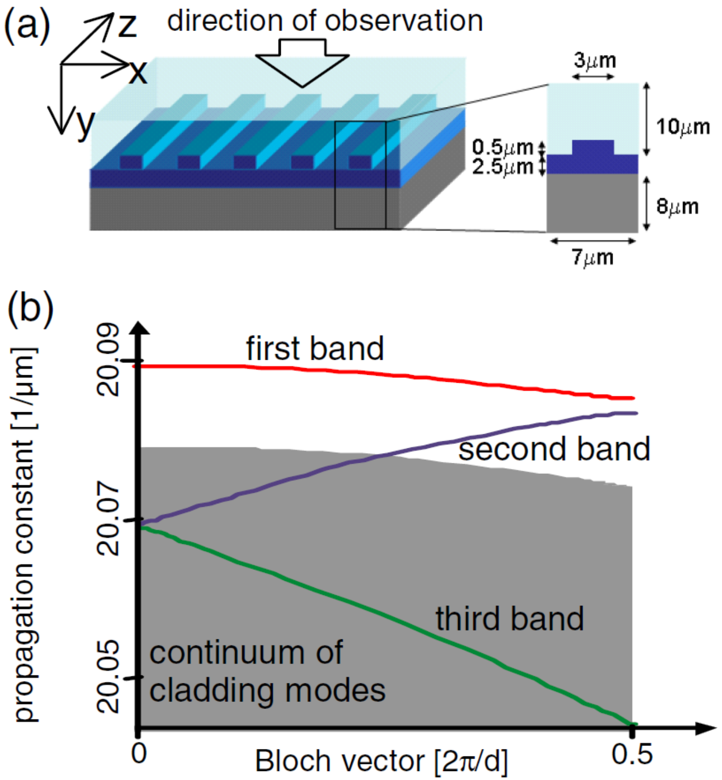
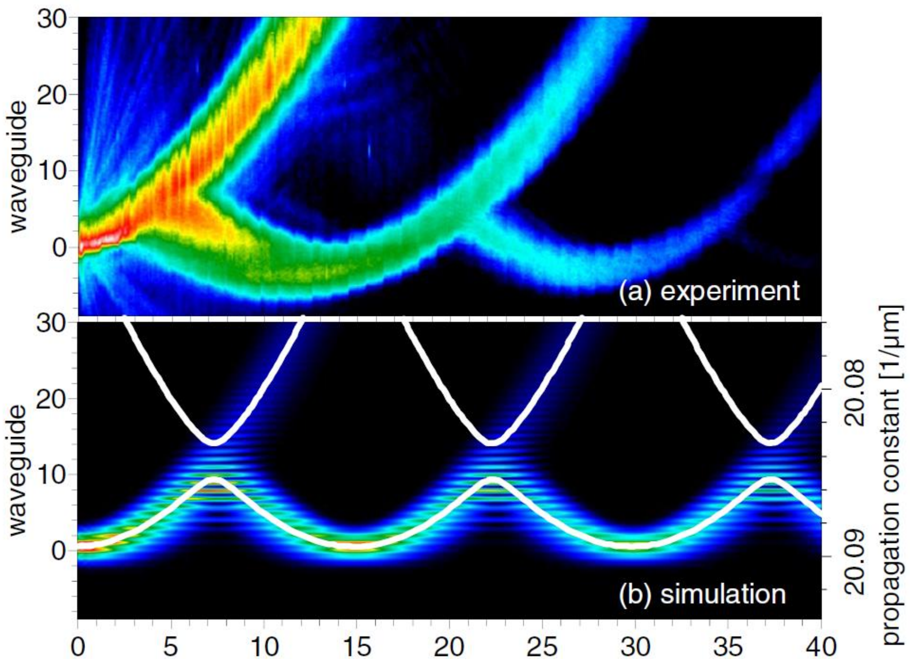
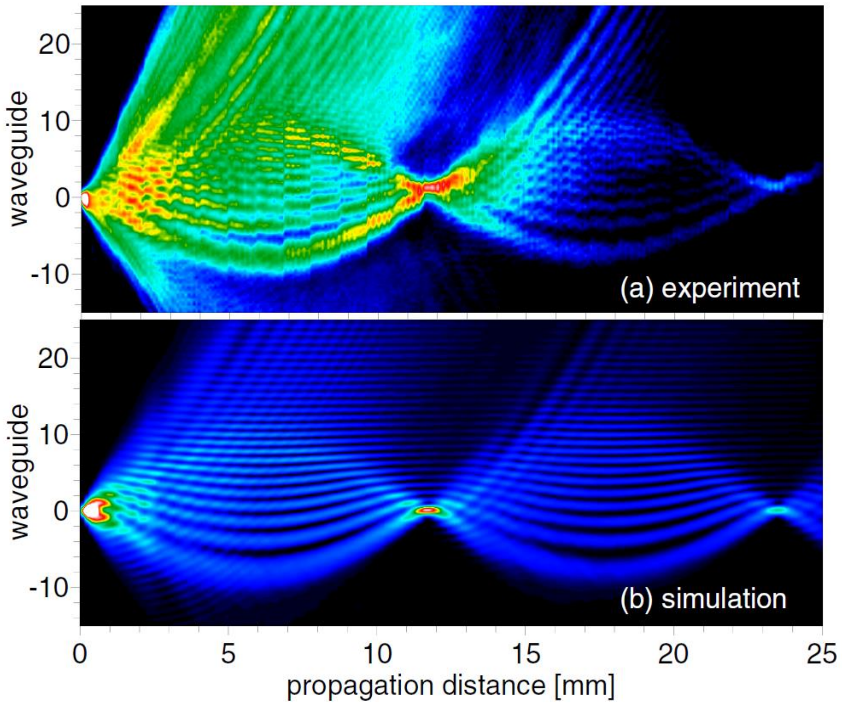
(Received 10 August 2005; published 18 January 2006; corrected 23 January 2006)







PRL 94, 113904 (2005) PHYSICAL REVIEW LETTERS week ending 25 MARCH 2005

Nonadiabatic Landau-Zener Tunneling in Waveguide Arrays with a Step in the Refractive Index

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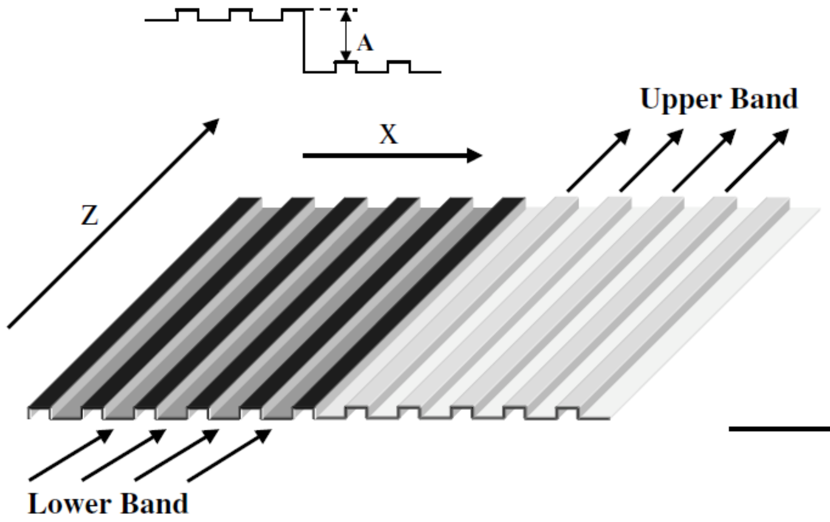
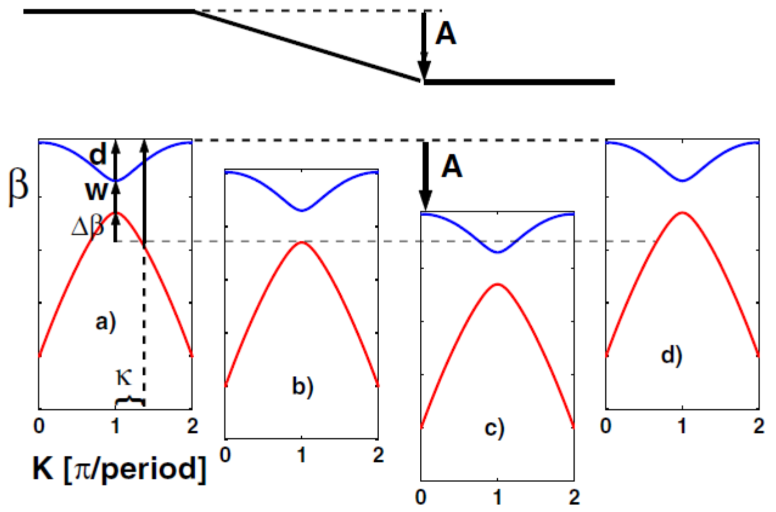
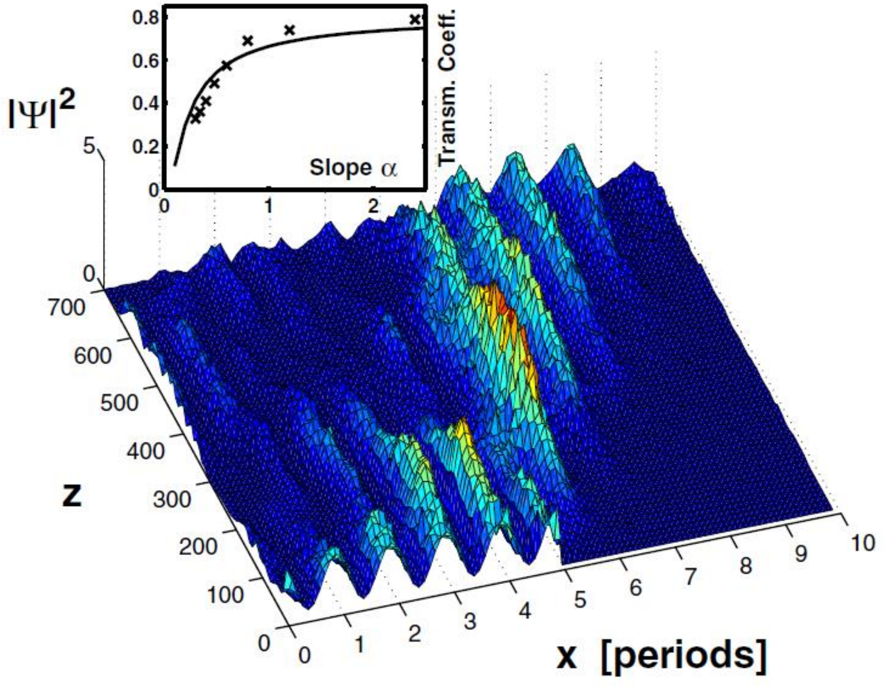
(Received 4 August 2004; published 23 March 2005)







Ferroelectric Vortex-Lattice Photonics