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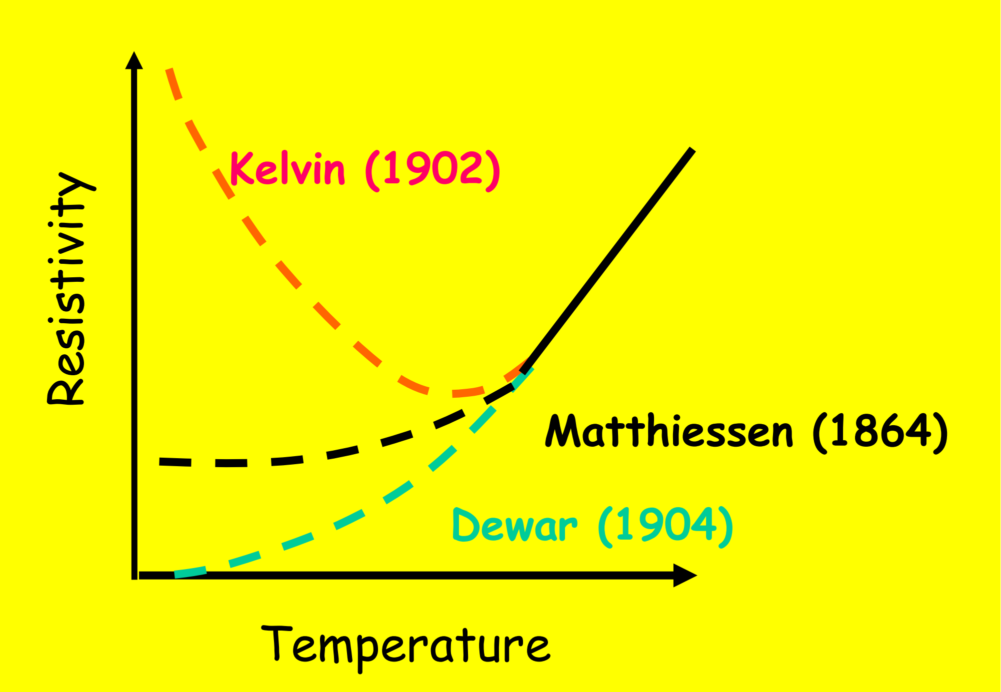
Superconductivity in nanostructures

Part I

Alexander Shengelaya

Batumi, ISCFMMT 2022, October 2022

Electrical resistivity at low temperatures



Kelvin: Electrons will be frozen – resistivity grows till ∞.

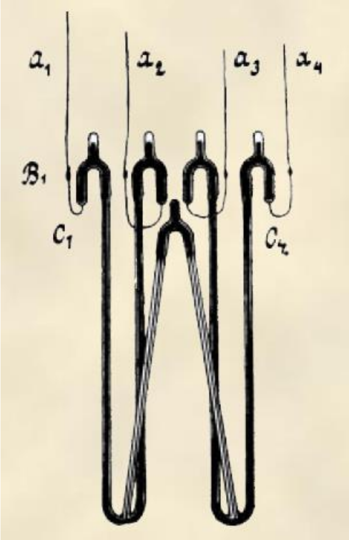
Dewar: the lattice will be frozen – the electrons will not be scattered. Resistivity wiil decrese till 0.

Matthiesen: Residual resistivity because of contamination and lattice defects.

One of the scientific challenges at the end of 19th and beginning of the 20th century: How to reach temperatures close to 0 K?

Hydrogen was liquefied (boiling point 20.28 K) for the first time by James Dewar in 1898

Superconductivity- discovery I



1895 William Ramsay in England discovered helium on the earth
1908 H. Kamerlingh Onnes liquefied helium (boiling point 4.22 K)

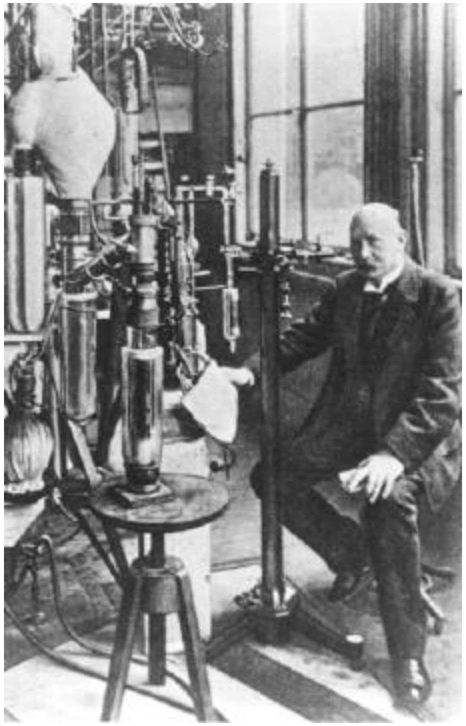
Resistivity at low temperatures- pure mercury (could repeatedly distilled producing very pure samples).

- Repeated resistivity measurements indicated zero resistance at the liquid-helium temperatures. **Short circuit was assumed!**
- During one repetitive experimental run, a young technician fall asleep. The helium pressure (kept below atmospheric one) slowly rose and, therefore, the boiling temperature. As it passed above 4.2 K, suddenly resistance appeared.

Hg T_C=4.2K

From: Rudolf de Bruyn Ouboter, "*Heike Kamerlingh Onnes's Discovery of Superconductivity*", Scientific American March 1997

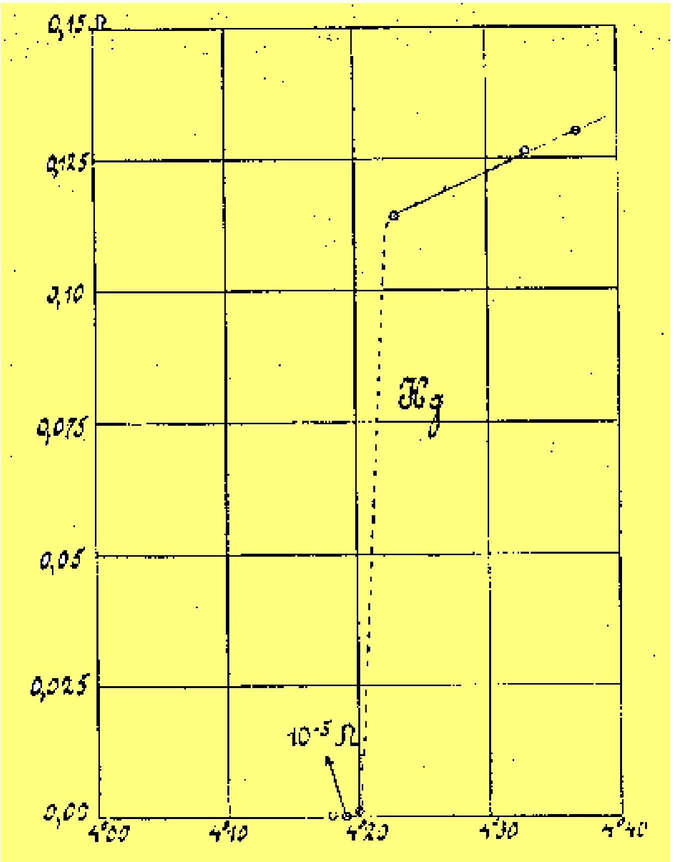
Superconductivity- discovery II



- Liquid Helium (4K) (1908). ***Boiling point*** 4.22K.
- Superconductivity in Hg T_C=4.2K (1911)

„Mercury has passed into a new state, which on account of its extraordinary electrical properties may be called the superconducting state"

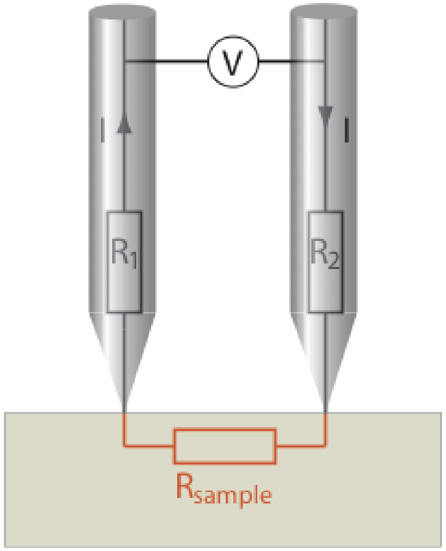
H. Kamerlingh Onnes 1913 (Nobel preis 1913)

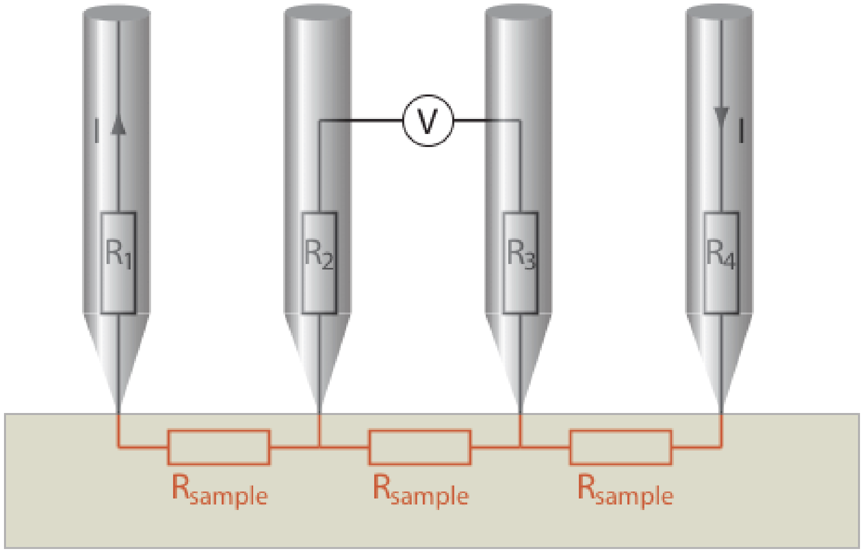


Resistivity R=0 below T_C; ($R<10^{-23}$ Ω·cm, 10^{18} times smaller than for Cu)

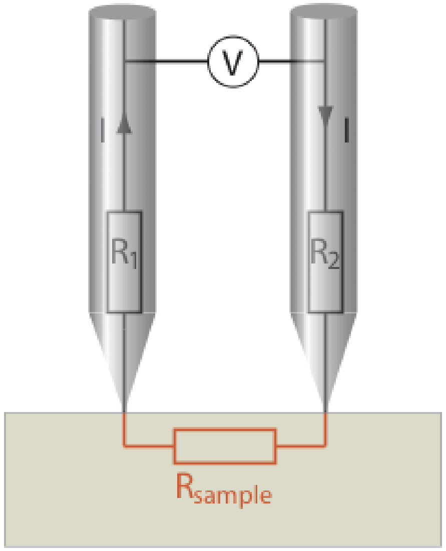
How to measure very small resistivity ?

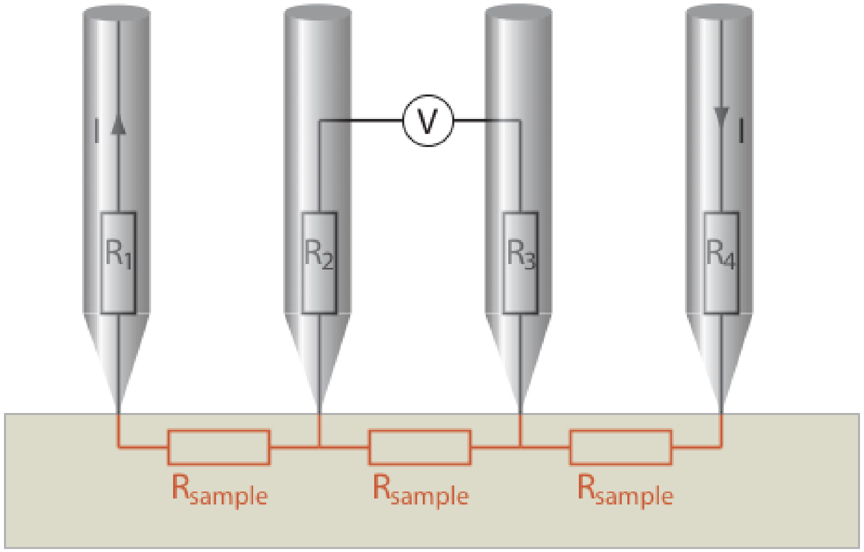
How to measure very small resistivity ?





Is the resistivity of a superconductor really zero or just very small?

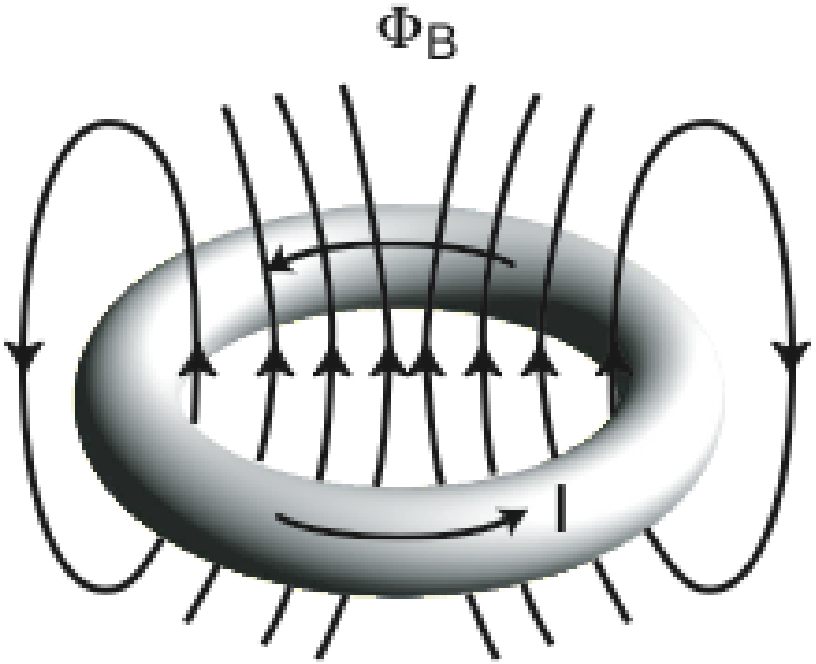




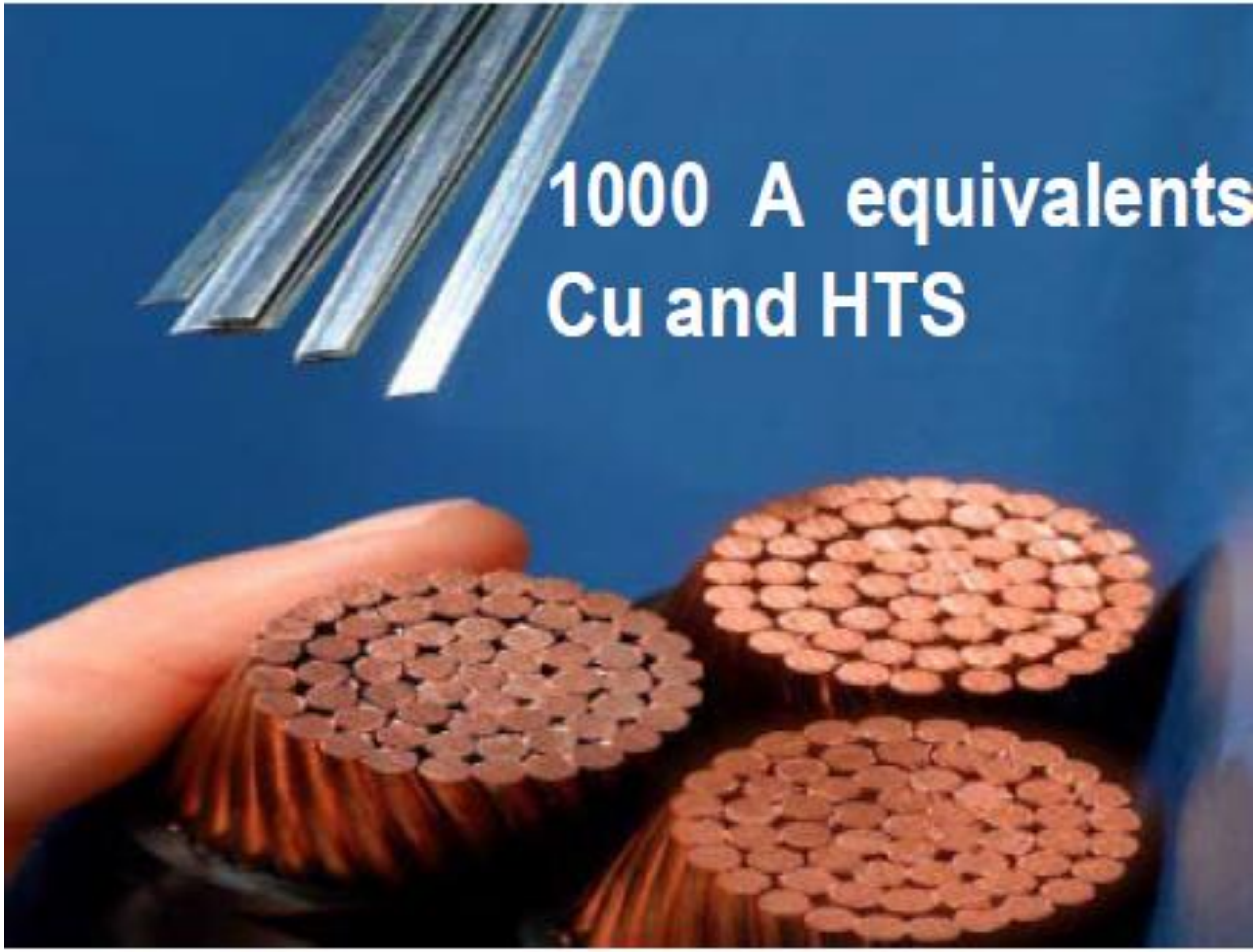
- A two-point probe measurement will not work because the the contact resistance and the resistance of the wires dominates everything.
- A four point probe measurement can be tried but it does not work either: one is limited by the smallest voltage one can measure.

Is the resistivity of a superconductor really zero or just very small?

$$I(t) = I_0 e^{-t/\tau}$$



- A super-current can be induced in a superconducting ring.
- The decay of the current is given by the relaxation time (10^{-14} s for a normal metal). For a superconductor it should be infinite. Experiments suggest that it is not less than 100.000 years.

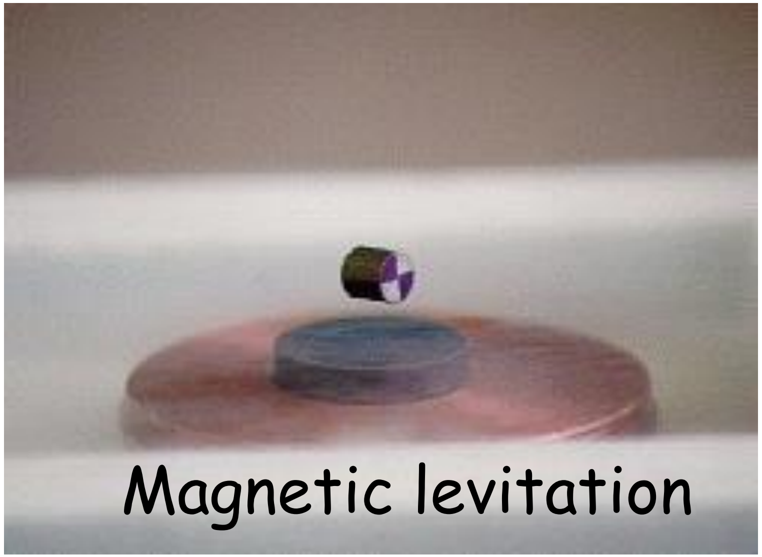

1000 A equivalents
Cu and HTS

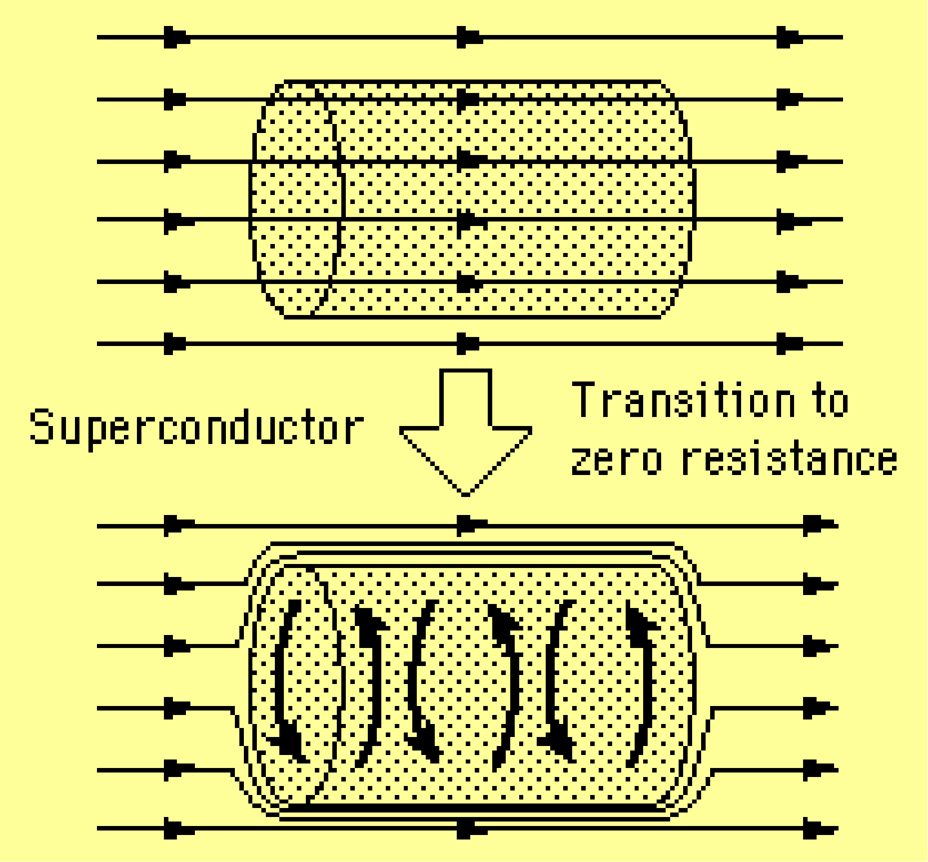
Meissner-Ochsenfeld-effect

A superconductor is a perfect diamagnet. Superconducting material expels magnetic flux from the interior.

W. Meissner, R. Ochsenfeld (1933)

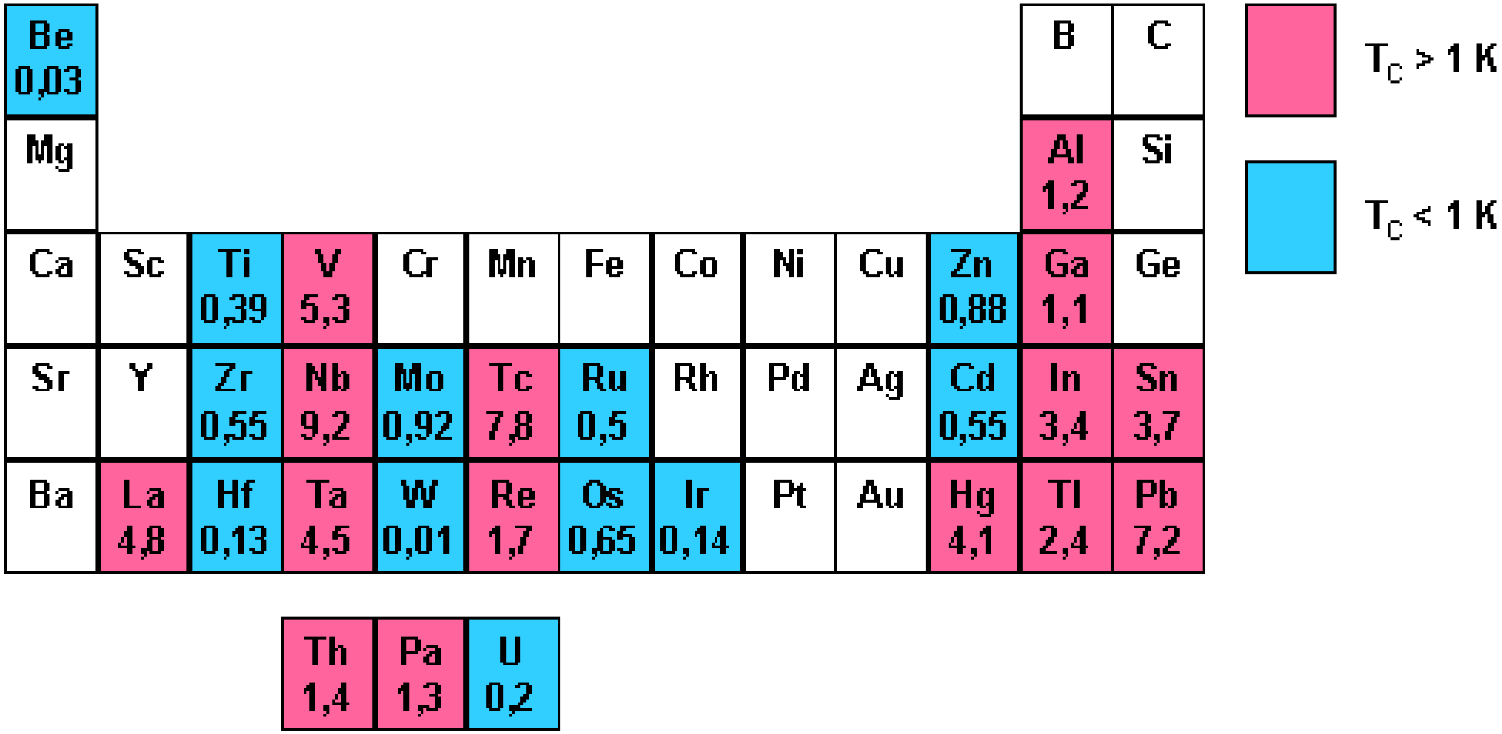
On the surface of a superconductor ($T<T_C$) superconducting current will be induced. This creates a magnetic field compensating the outside one.


Magnetic levitation



Screening (shielding) currents

Superconducting elements



Be 0,03											B	C
Mg											Al 1,2	Si
Ca	Sc	Ti 0,39	V 5,3	Cr	Mn	Fe	Co	Ni	Cu	Zn 0,88	Ga 1,1	Ge
Sr	Y	Zr 0,55	Nb 9,2	Mo 0,92	Tc 7,8	Ru 0,5	Rh	Pd	Ag	Cd 0,55	In 3,4	Sn 3,7
Ba	La 4,8	Hf 0,13	Ta 4,5	W 0,01	Re 1,7	Os 0,65	Ir 0,14	Pt	Au	Hg 4,1	Tl 2,4	Pb 7,2
			Th 1,4	Pa 1,3	U 0,2							

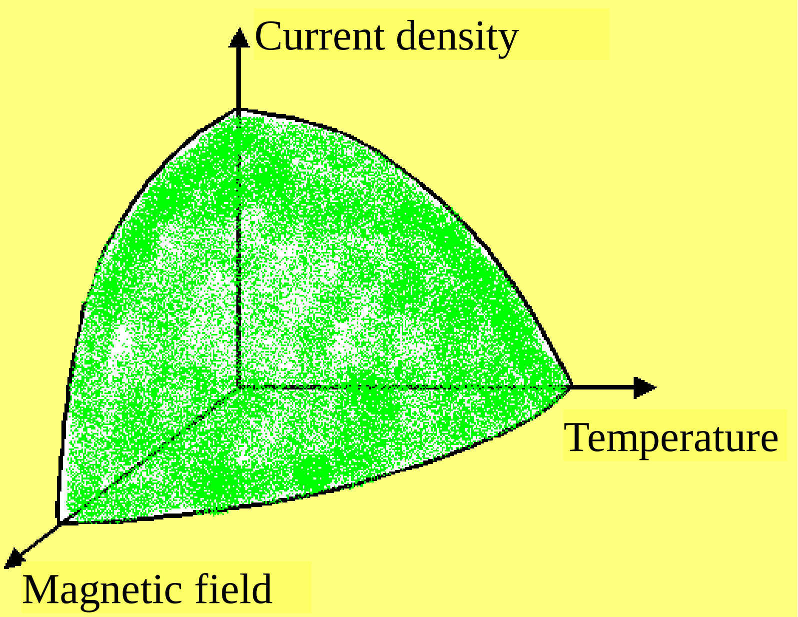
🟥 $T_C > 1$ K

🟦 $T_C < 1$ K

- Ferromagnetic elements are not superconducting
- The best conductors (Ag, Cu, Au..) are not superconducting
- Nb has the highest T_C = 9.2K from all the elements

What destroys superconductivity?

A current: produces magnetic field which in turn destroys superconductivity.



High temperatures: strong thermal vibration of the lattice predominate over the electron-phonon coupling.

Magnetic field: the spins of the C-P will be directed parallel.

(should be antiparallel in C-P)

High Temperature Superconductivity

The Nobel Prize in Physics 1987

"for their important break-through in the discovery of superconductivity in ceramic materials"

J. Georg Bednorz

K. Alexander Müller

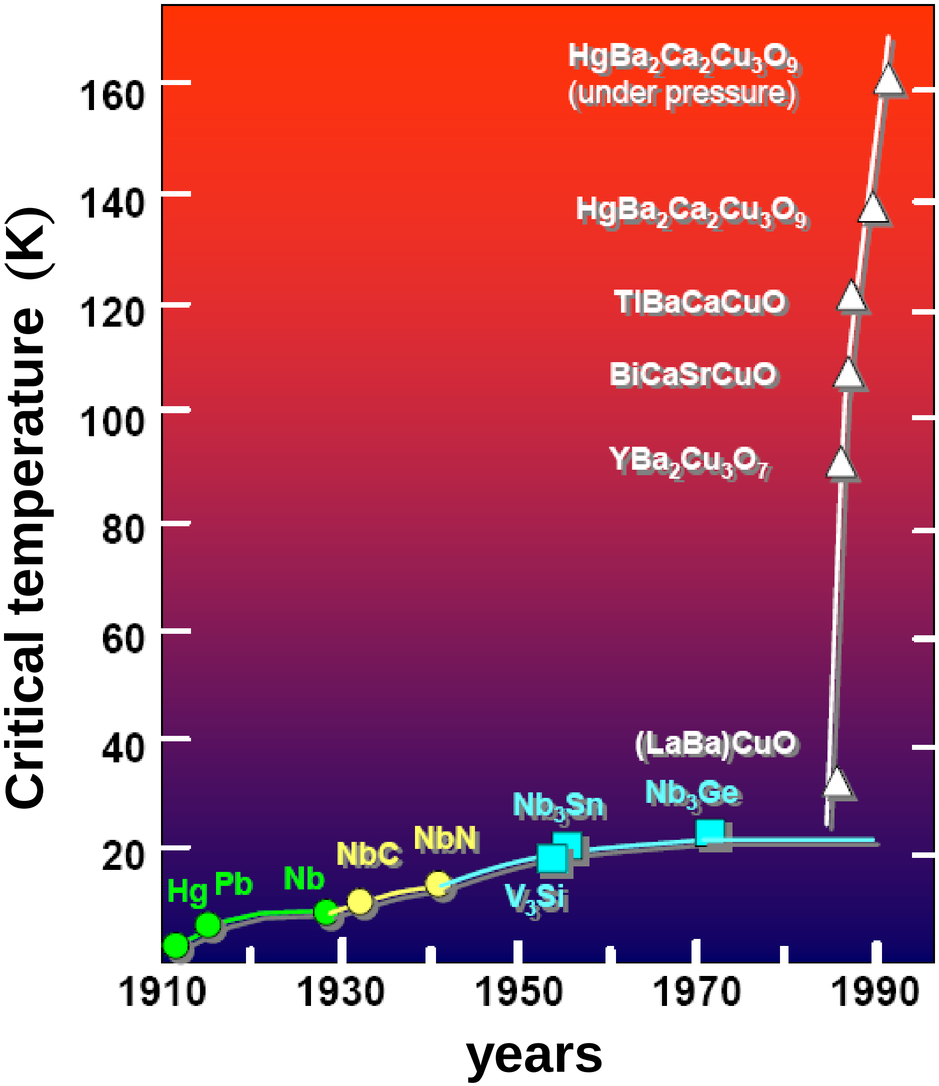
$(La_{1.85}Ba_{.15})CuO_4$

$YBa_2Cu_3O_7$

$Bi_2Sr_2CaCu_2O_8$

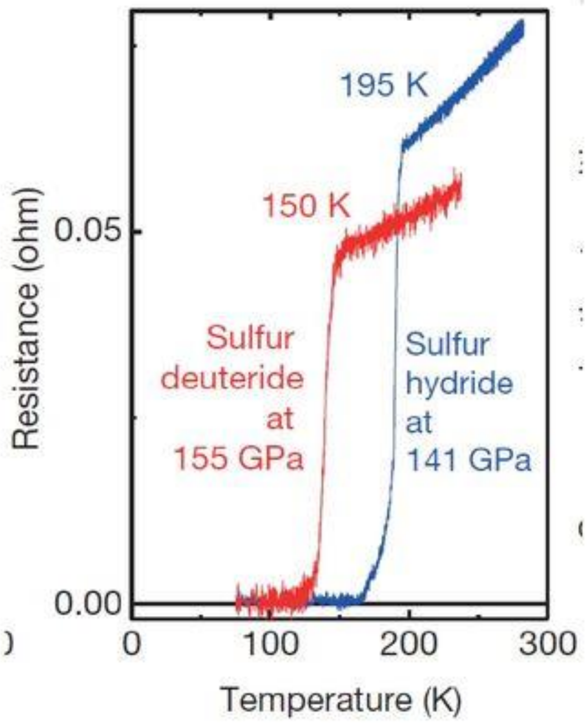
$Tl_2Ba_2Ca_2Cu_3O_{10}$

$Hg_{0.8}Tl_{0.2}Ba_2Ca_2Cu_3O_{8.33}$

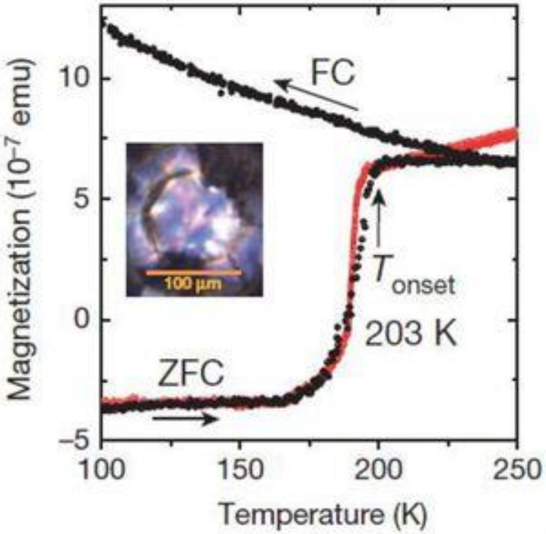


Tc = 203 K Superconductivity in Sulfur Hydride (H_3S) @ P=200 GPa

Resistance

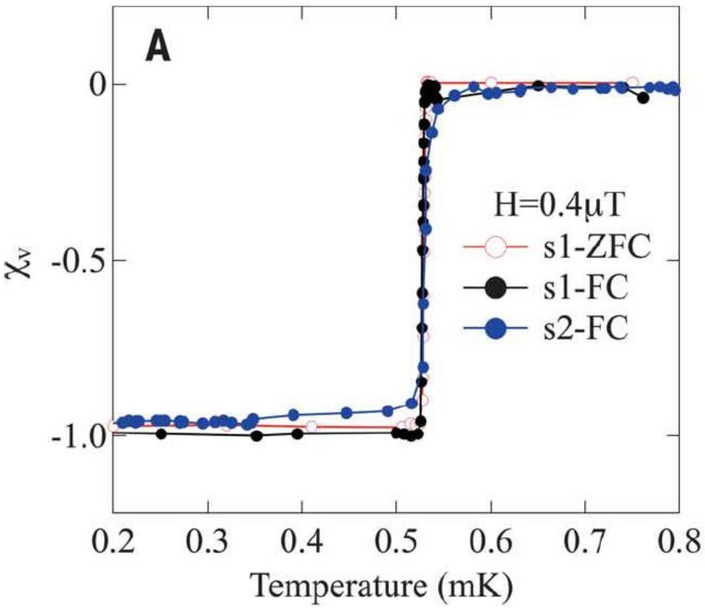


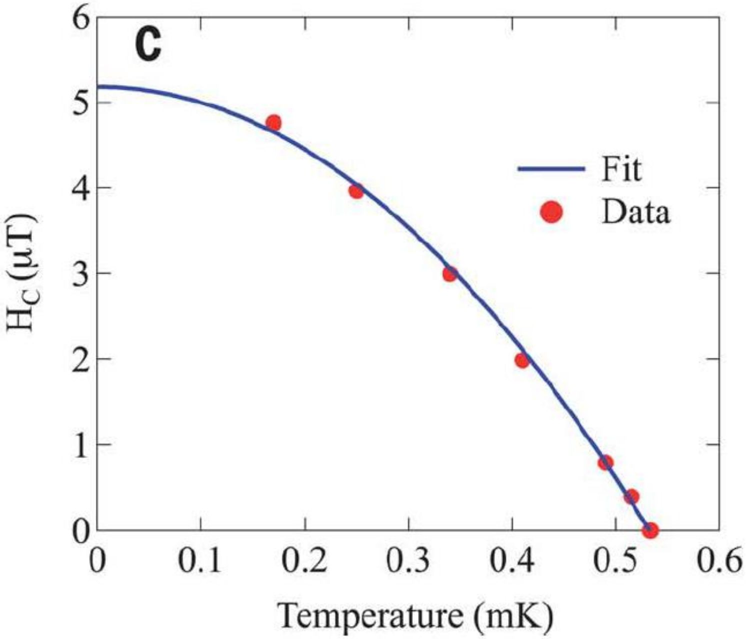
Magnetization/ Meissner



A.P. Drozdov, M.I. Eremets, I.A. Troyan, V. Ksenofontv & S.I. Shylin, Nature **525**, 73 (2015).

Superconductivity in pure Bi at ambient pressure





Prakash *et al.*, *Science* **355**, 52–55 (2017)

Magnetic shielding

Special alloy: **mumetal**

Very high magnetic permeability $\mu \approx 100000$

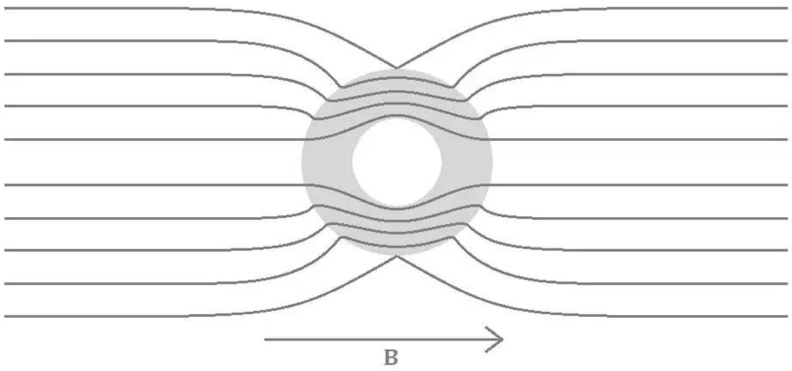
77 % Ni, 16 % Fe, 5 % Cu, 2 % Cr or Mo

Shielding value $$S = \mu \frac{d}{D}$$

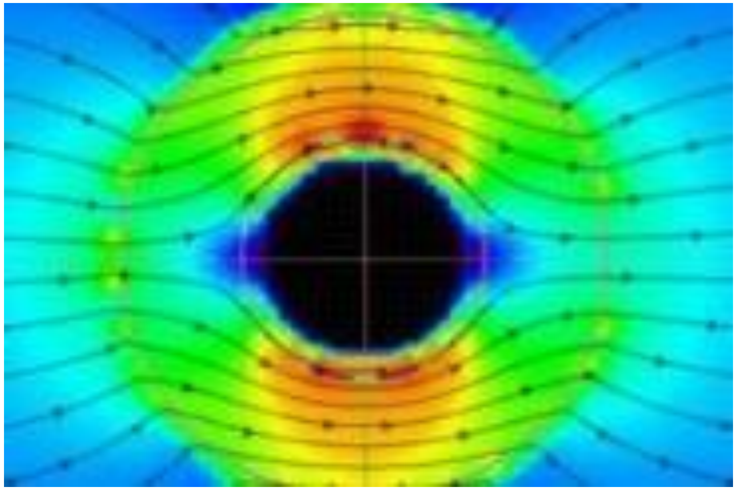
d : materials thickness

D : shielding diameter

Magnetic shielding

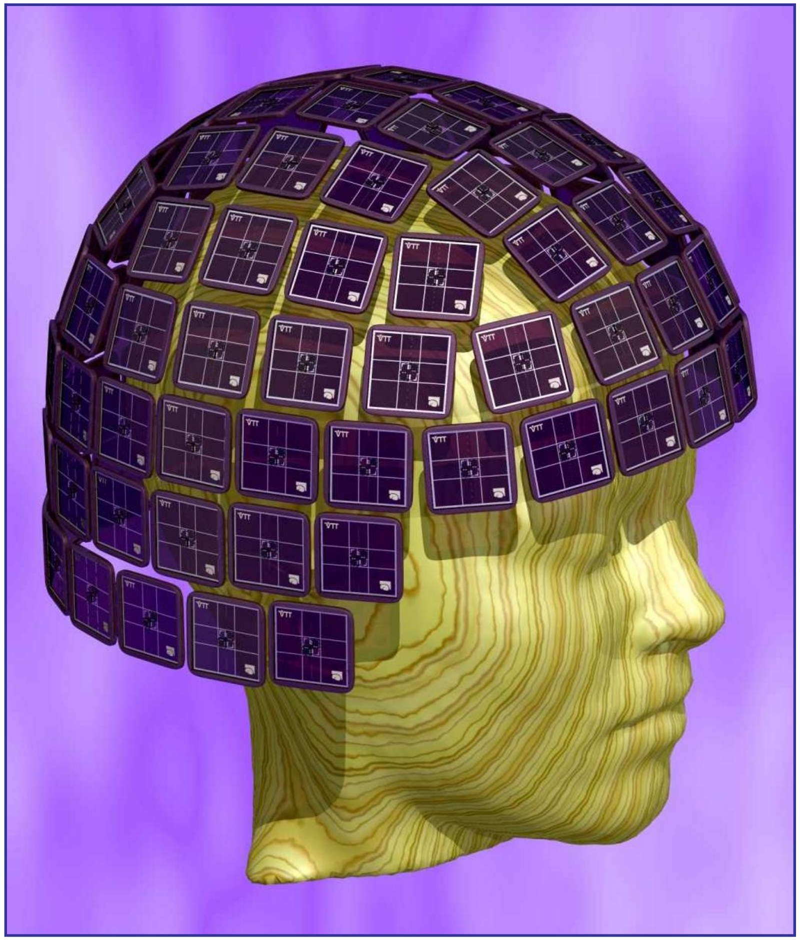


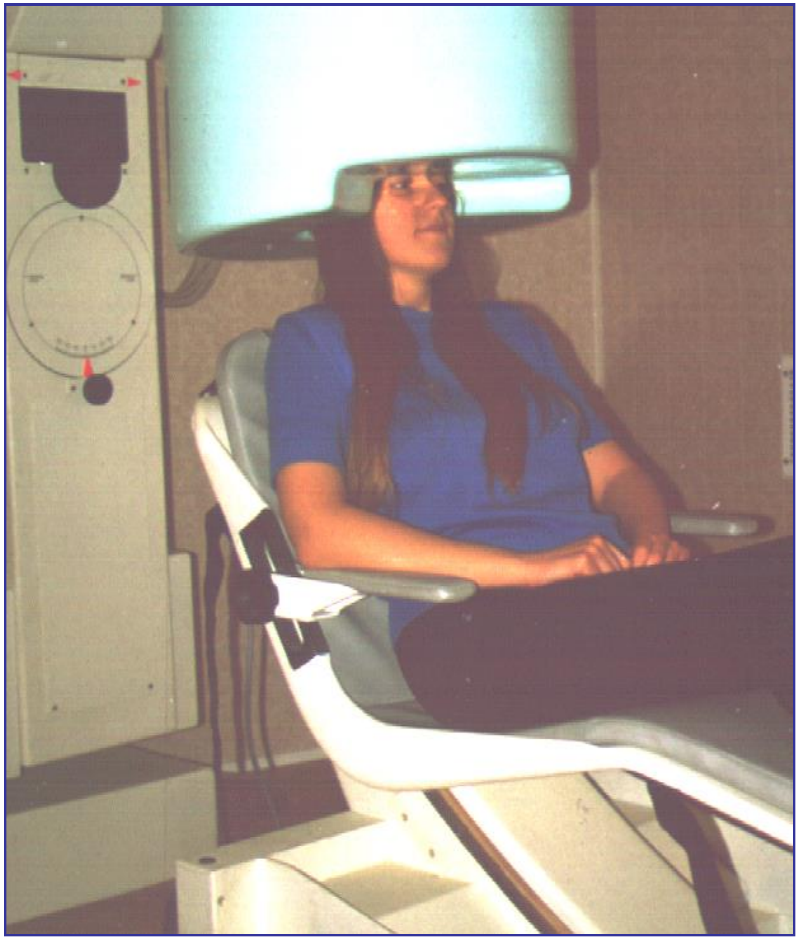
https://mumetal.co.uk/?p=106



https://magneticshields.co.uk/

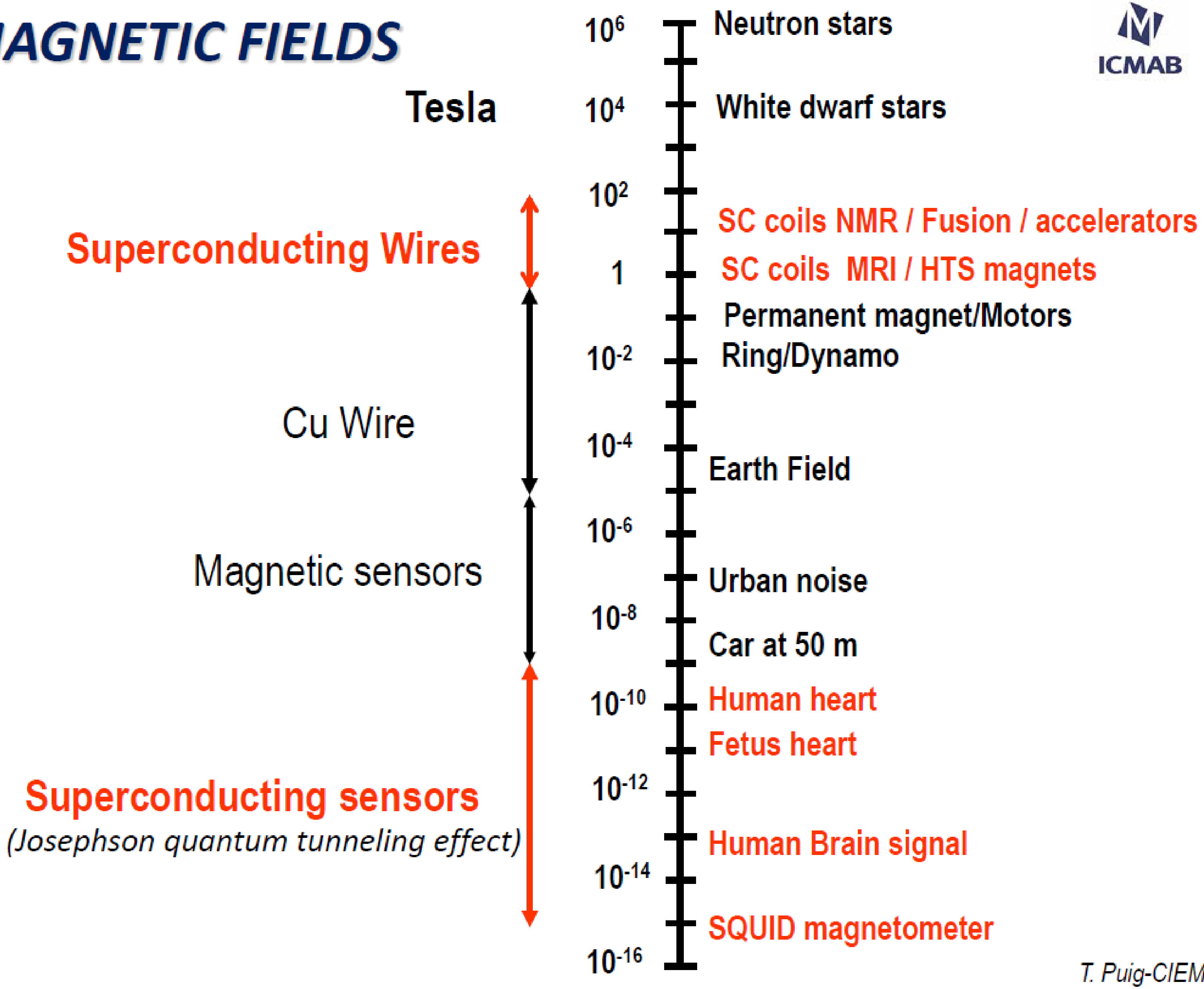
Detecting 100 fT magnetic fields using SQUIDs, Superconducting Quantum Interference Devices





Prototype of a SQUID-based magnetoencelography system developed at the Chieti University (courtesy of G.L. Romani).

MAGNETIC FIELDS



Tesla

10^{6} **Neutron stars**

10^{4} **White dwarf stars**

10^{2}

SC coils NMR / Fusion / accelerators

1 **SC coils MRI / HTS magnets**

Permanent magnet/Motors

10^{-2} **Ring/Dynamo**

10^{-4}

Earth Field

10^{-6}

Urban noise

10^{-8}

Car at 50 m

10^{-10} **Human heart**

Fetus heart

10^{-12}

Human Brain signal

10^{-14}

SQUID magnetometer

10^{-16}

Superconducting Wires

Cu Wire

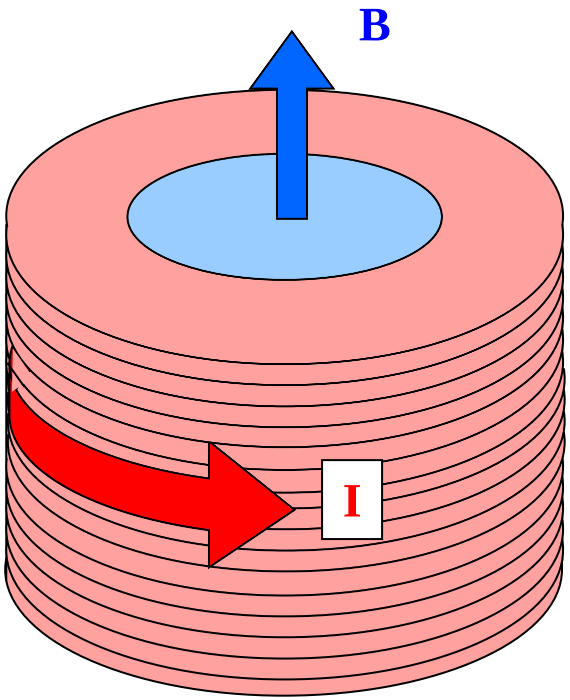
Magnetic sensors

Superconducting sensors

(Josephson quantum tunneling effect)

T. Puig-CIEMAT2016

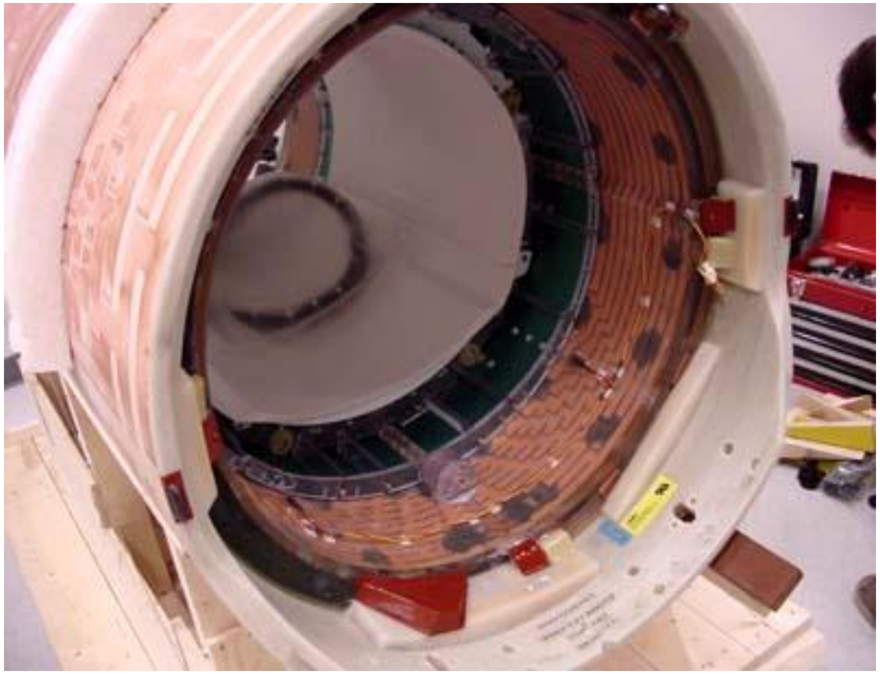
Creating strong magnetic fields



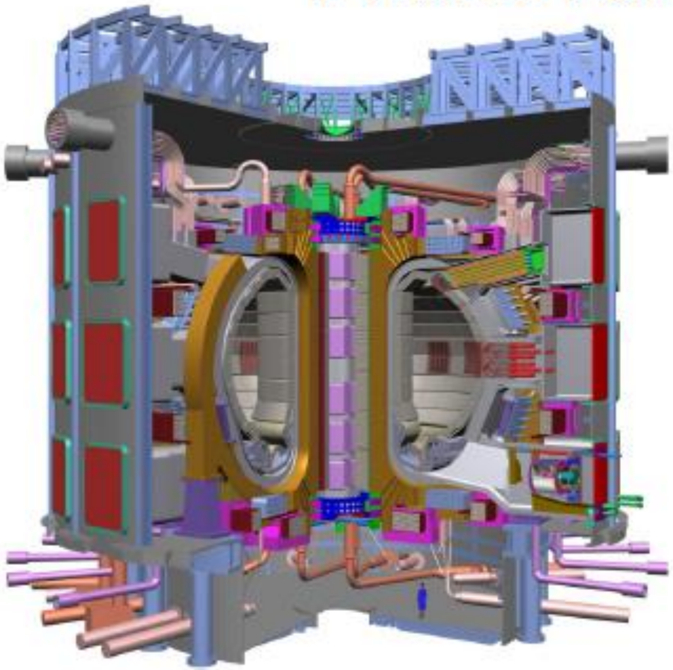
Solenoid

მაგნეტორეზონანსული ტომოგრაფია (MRI) ზეგამტარობის გამოყენება მედიცინაში

მაგნეტორეზონანსული ტომოგრაფია (MRI)
ზეგამტარობის გამოყენება მედიცინაში



FUSION and Superconductivity



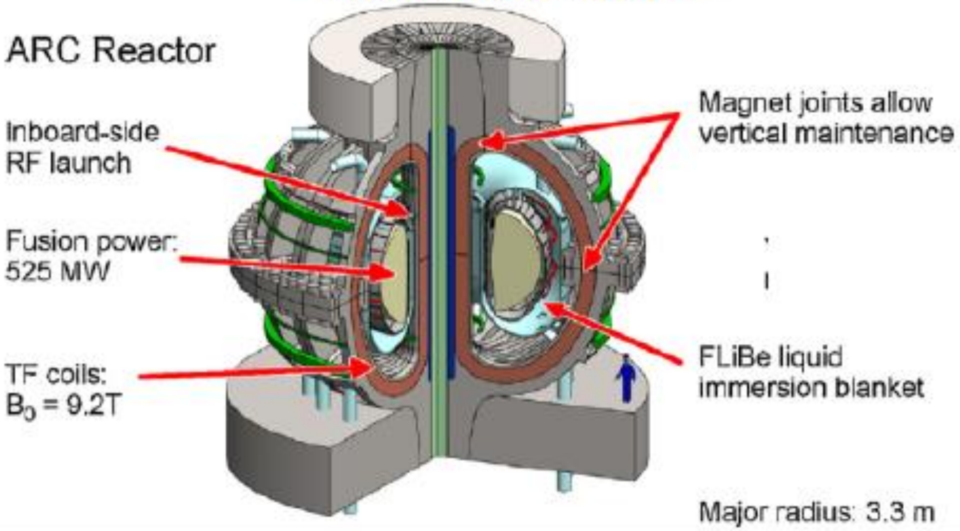
ITER is example of previous state-of-art in superconductor magnets

- Very large size
- > 2 years to replace interior shield
- H=10 T, coils diameter=12 m
- 500 MW fusion power

ARC is example of using new HTS magnets (Affordable, Reliable, Compact)

- Decrease volume by factor of ten!
- Modular replacement of interior
- New HTS magnet at ~20 T, 20 K
- 500 MW fusion power

NEXT **DEMO**:
2000-4000 MW

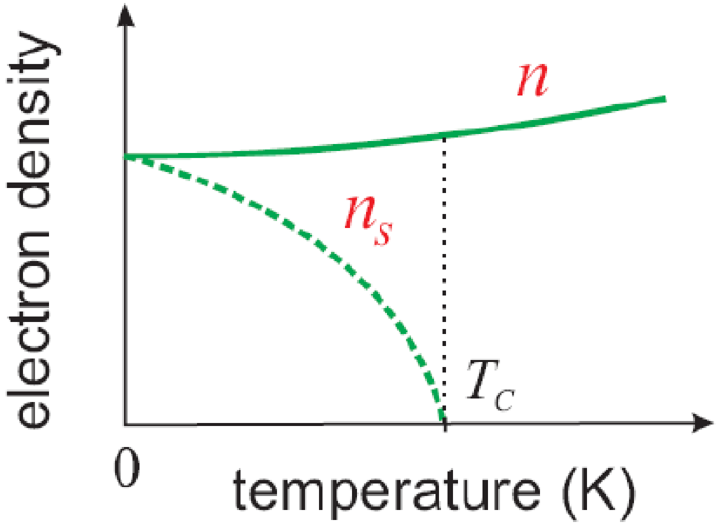


T. Puig -CIEMAT 2016

London model (Two fluid model)

Heinz und Fritz London

- all free electrons of the superconductor are divided into two groups:
  - superconducting electrons of density n_{s}
  - normal electrons of density n_{n}.
- The total density of free electrons is $n = n_{\mathrm{s}} + n_{\mathrm{n}}$
- As the temperature increases from 0 to T_c, the density n_{s} decreases from n to 0.

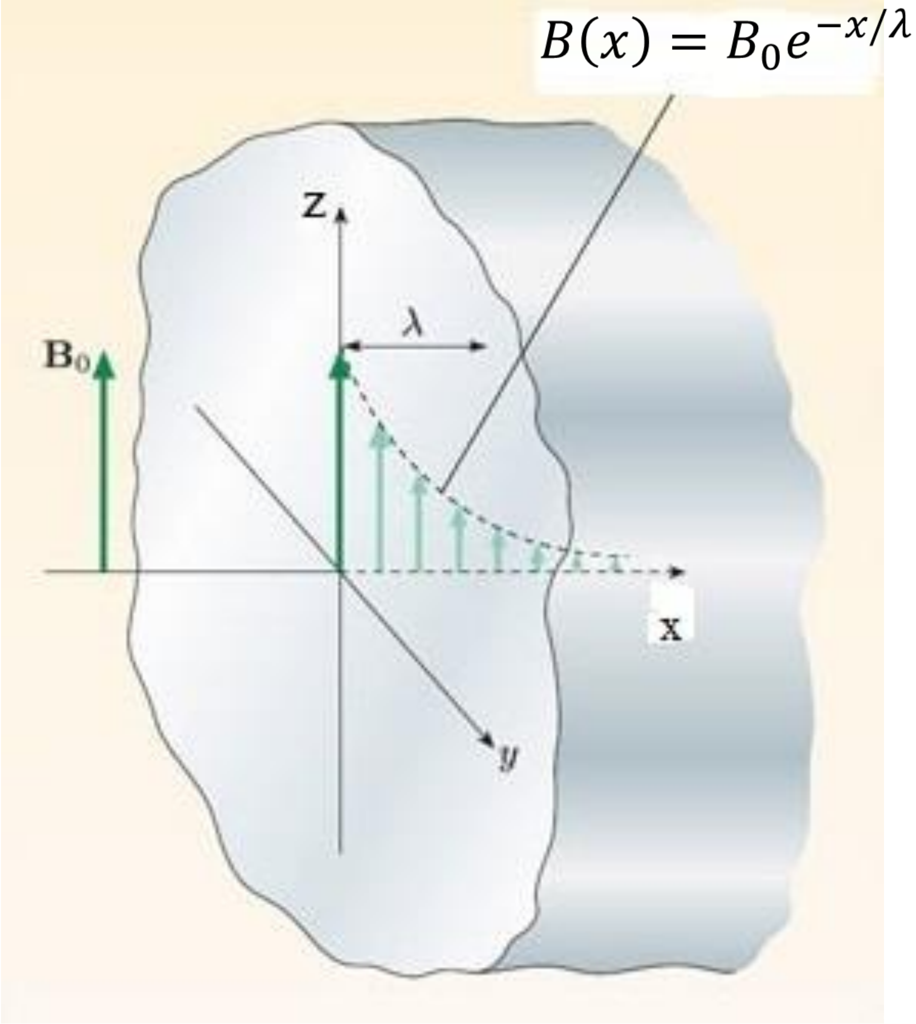


Magnetic field penetration depth λ

$$\boxed{\vec{\nabla}^2 \vec{B} = \frac{1}{\lambda^2} \vec{B}}$$

$$\lambda^2 = \frac{m}{\mu_0 n_s e^2}$$

$$B(x) = B_0 e^{-x/\lambda}$$

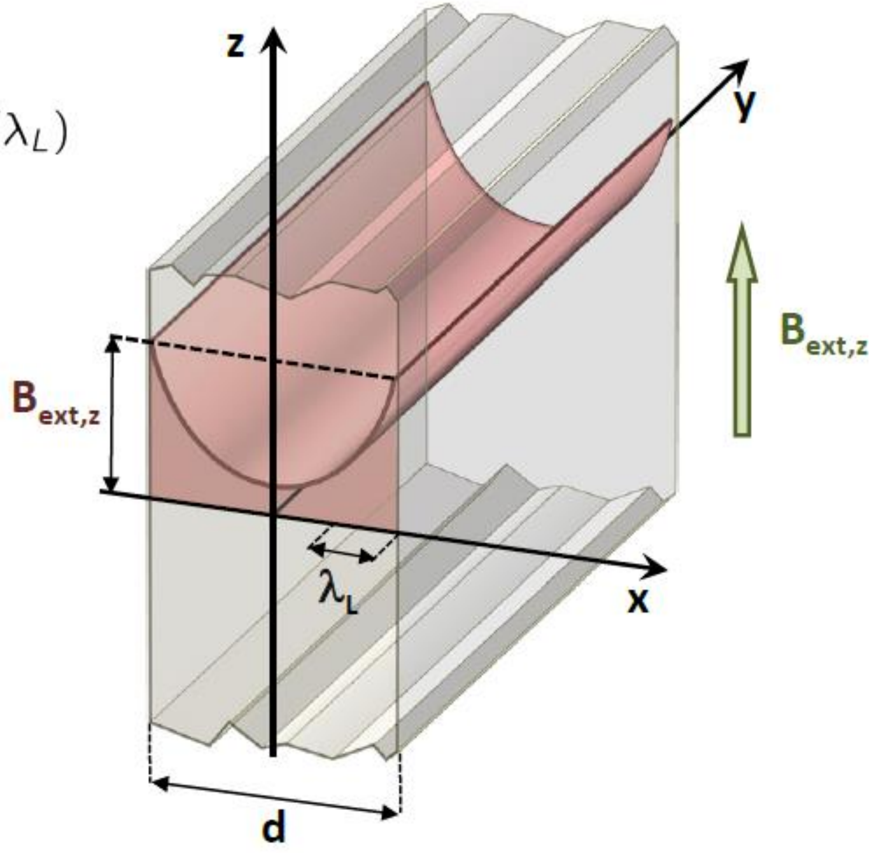


- example: thin superconducting sheet of thickness *d* with *B* || sheet
- Ansatz:

$$B_z(x) = B_{\text{ext,z}} \exp(-x/\lambda_L) + B_{\text{ext,z}} \exp(+x/\lambda_L)$$

- boundary conditions:

$$B_z(-d/2) = B_z(+d/2) = B_{\text{ext,z}}$$



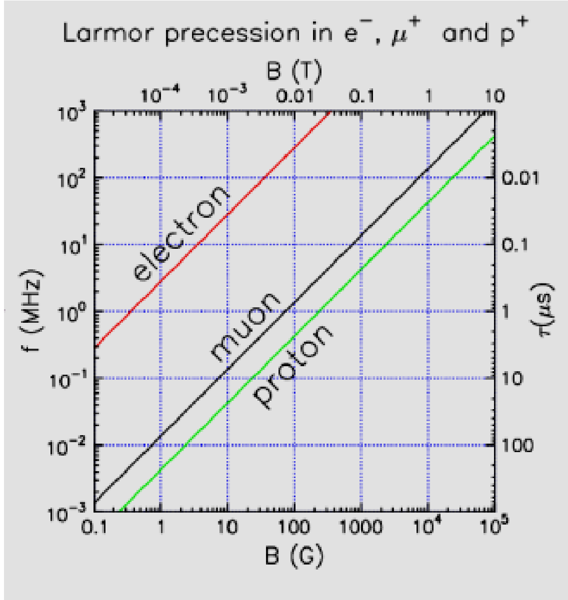
Estimate magnetic field penetration depth λ

$$\lambda^2 = \frac{m}{\mu_0 n_s e^2}$$

Muon precession

- beam of spin polarized muons μ^+ with polarization $\boldsymbol{P}_\mu(0)$
- μ^+ precesses around the local field with Larmor frequency ω_μ

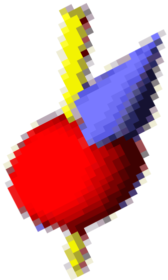
local magnetic field



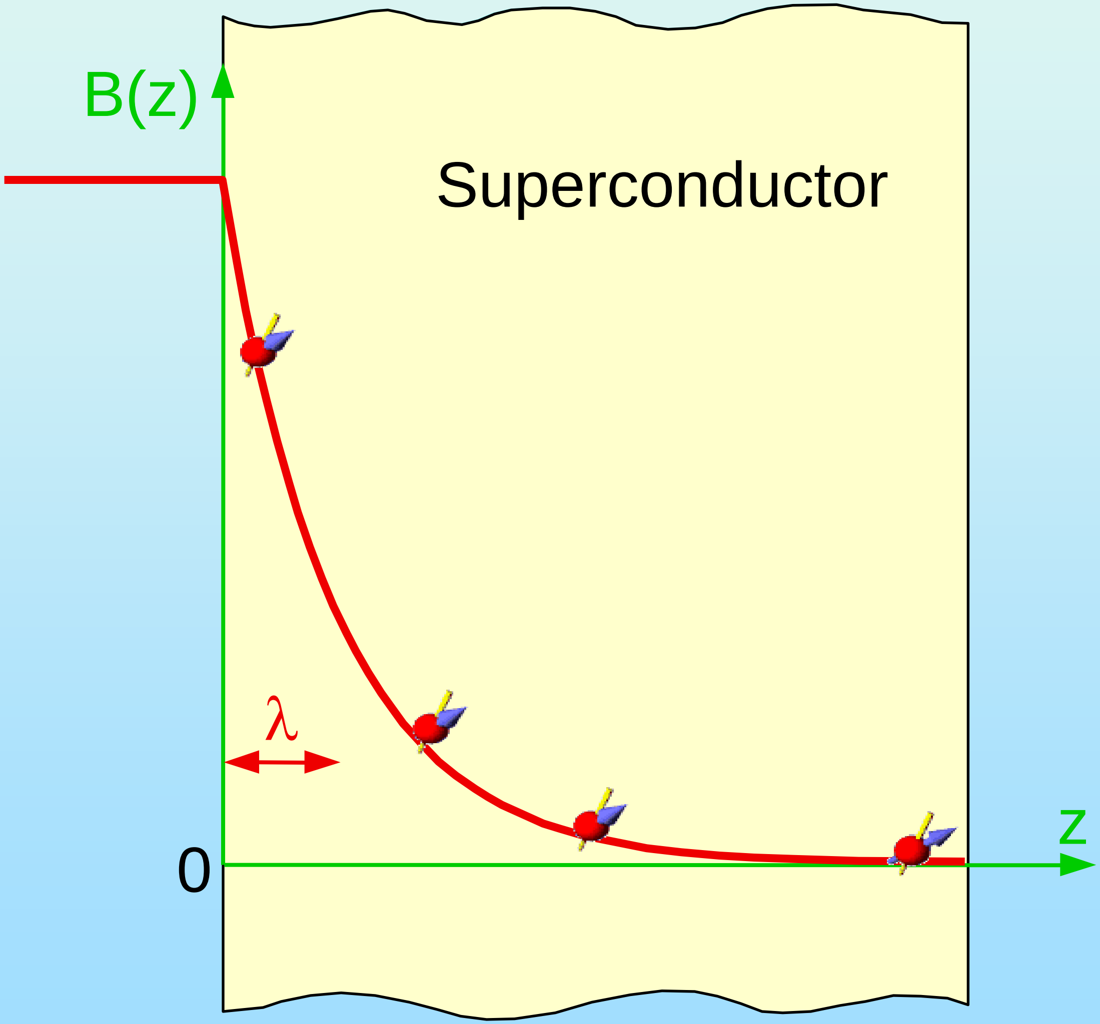
$$\boxed{\omega_\mu = \gamma_\mu B_{loc}}$$

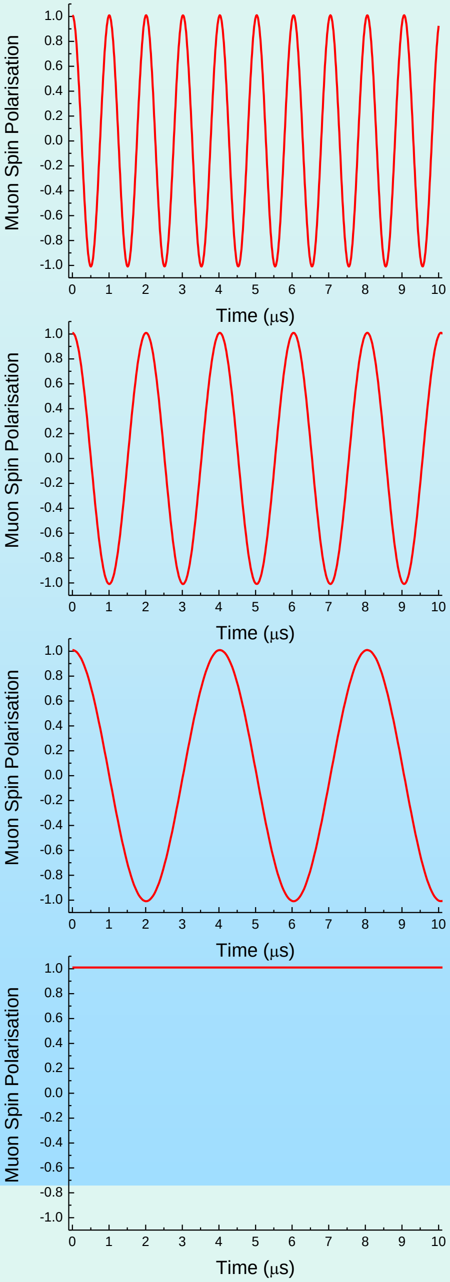
gyromagnetic ratio

γ_μ= 135.5 MHz/T



Depth dependent μSR measurements in near surface regions



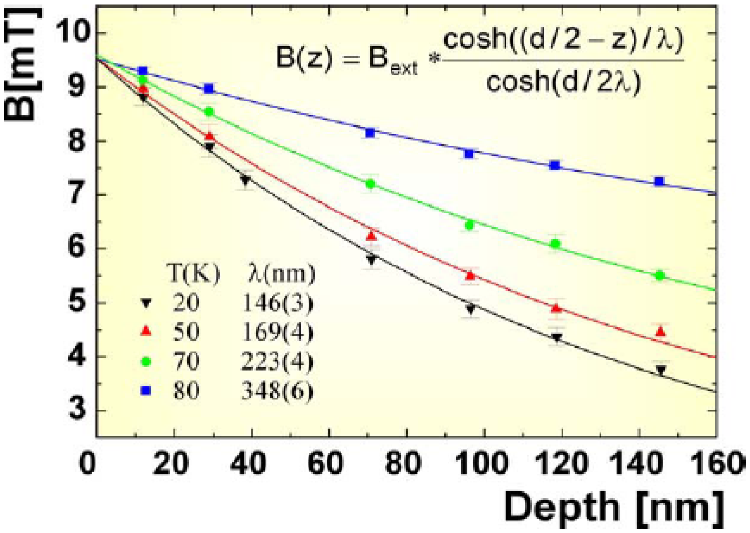


→ Magnetic field profile B(z) over nm scale

→ Characteristic lengths of the sc λ, ξ

⇒ B(z)

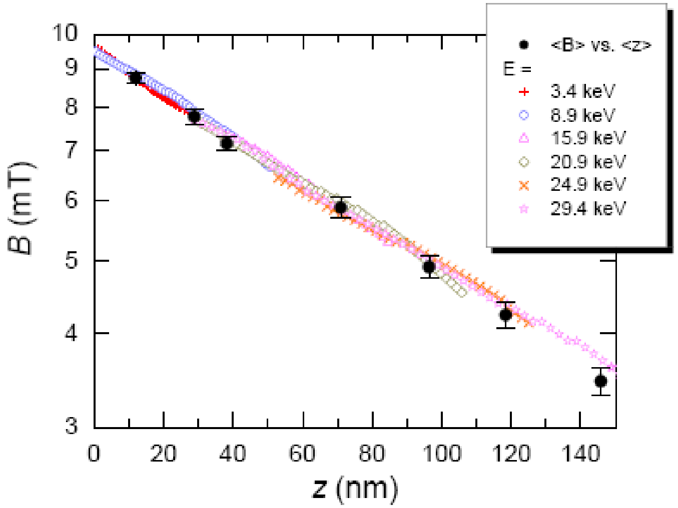
Field profile at the surface of $YBa_2Cu_3O_{7-\delta}$

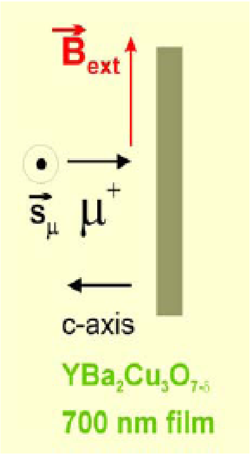


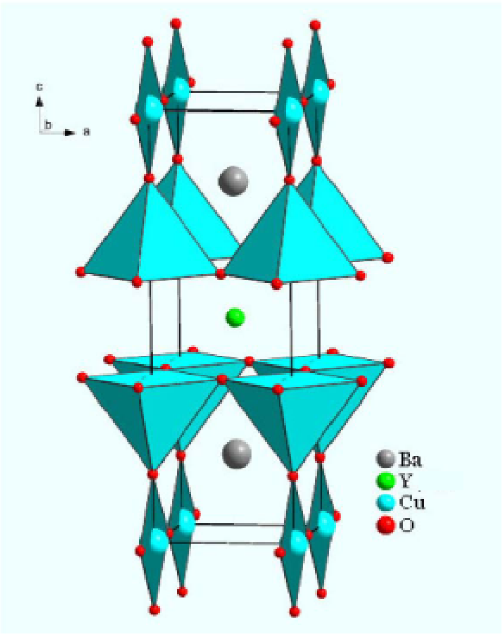
First direct measurements of field profile on nm scale. Microscopic confirmation of London equations (local electrodynamics). **Absolute determination** of magnetic penetration depth (λ_{ab}) **without knowledge of superconducting state**.

$$B(z,T)) = B_{ext} e^{-\frac{z}{\lambda_{ab}(T)}}$$

Field decay determined by shielding current flowing in **ab** planes → λ_{ab}







T.J. Jackson, T.M. Riseman, E.M. Forgan, H. Glückler, T. Prokscha, E. Morenzoni, M. Pleines, Ch. Niedermayer, G. Schatz, H. Luetkens, and J. Litterst, Phys. Rev. Lett. **84**, 4958 (2000).

	T_c	λ [nm]	ξ[nm]	κ
Al	1.2	16	1600	0.01
Sn	3.7	34	230	0.16
Pb	7.2	37	83	0.4

	T_c	λ [nm]	ξ[nm]	κ
Nb	9.3	39	38	1
Nb_3Sn	18	80	3	27
$YBa_2Cu_3O_7$	93	150	1.5	100
Rb_3C_{60}	30	247	2.0	124
$Bi_2Sr_2Ca_2Cu_3O_{10}$	110	200	1.4	143

Ginzburg-Landau theory (1950)

order parameter

$$\Psi(\mathbf{r}) = \Psi_0 e^{i\phi(\mathbf{r})}$$

$$n_s = |\Psi^* \Psi| = \Psi_0^2$$

The Nobel Prize in Physics 1972

"for their jointly developed theory of superconductivity, usually called the BCS-theory"

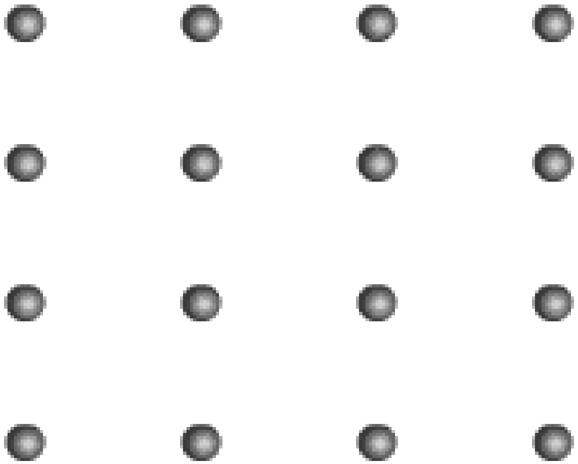
John Bardeen

Leon Neil Cooper

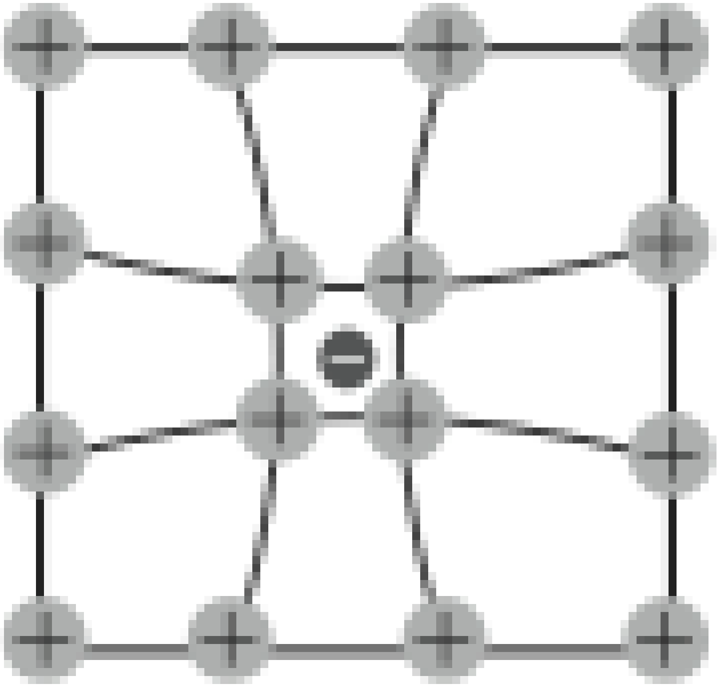
John Robert Schrieffer

Mechanism of superconductivity: pairing of electrons (Cooper pairs) due to interaction with lattice

(Electron-Phonon interaction).

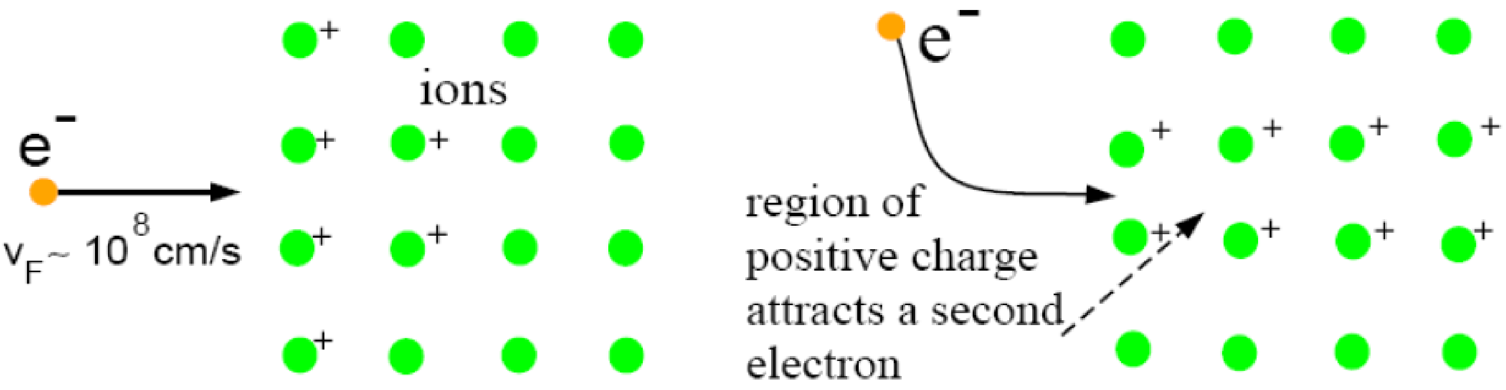


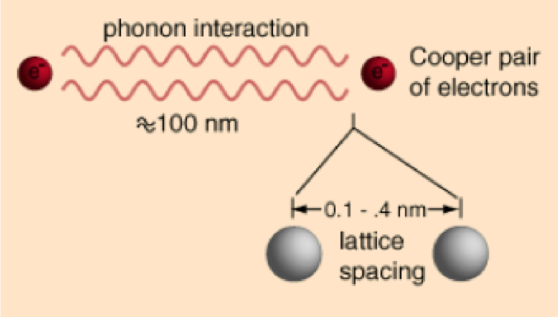
The electron-phonon interaction/ Cooper pairs



- Polarization of the lattice by one electron leads to an attractive potential for another electron.

BCS: attractive e^--e^- interaction



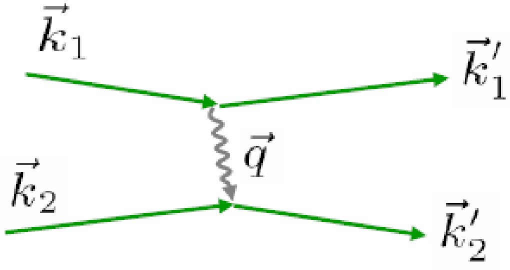


Lattice deforms slowly in the time of the electron

Maximum deformation of lattice at time $\tau \sim \dfrac{2\pi}{\omega_D} \sim 10^{-13}\,s$

In this time, first electron has travelled $\sim v_F \tau \sim 10^6\,ms^{-1} \times 10^{-13}\,s \sim 1000\,\text{Å}$

Lattice deformation attracts 2nd electron without it feeling the electron feeling Coulomb repulsion of 1st



$\hbar\omega_{\vec{q}}$ phonon energy

electron–electron interaction via emission and subsequent absorption of a phonon of momentum $\hbar\vec{q}$

$$\vec{k}_1 = \vec{k}_1' + \vec{q} \quad \Rightarrow \quad \vec{k}_1 + \vec{k}_2 = \vec{k}_1' + \vec{k}_2'$$

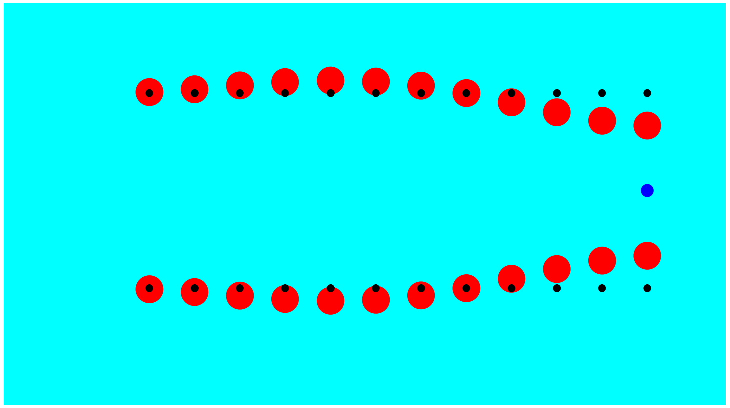
Scattering from state $\vec{k}_1$ to the state $\vec{k}_1'$ gives rise to local oscillations of electron density of frequency $\omega = (\overline{\varepsilon}_{\vec{k}1} - \overline{\varepsilon}_{\vec{k}1'})/\hbar$, where $\omega < \omega_{\mathrm{D}}$ (Debye frequency).

To enable an electron to go from the state $\vec{k}_1$ to the state $\vec{k}_1'$, the latter must be free (Pauli principle) that is possible only in the vicinity of the Fermi surface.

BCS theory of superconductivity

1957 John Bardeen, Leon Cooper, and John Robert Schrieffer

An electron on the way through the lattice interacts with lattice sites (cations). The electron produces **phonon**.

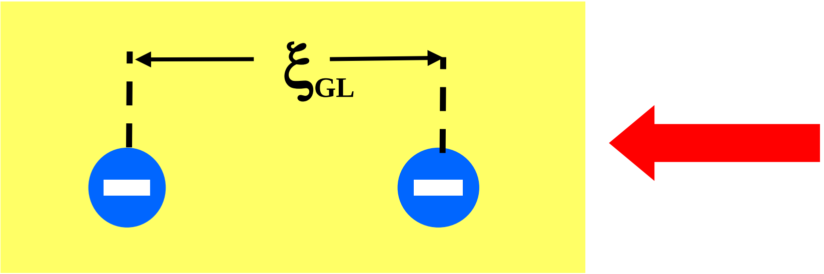


During one phonon oscillation an electron can cover a distance of ~10^4Å. The second electron will be attracted without experiencing the repulsing electrostatic force .

The lattice deformation creates a region of relative positive charge which can attract another electron.

Important length scales in a superconductor

Coherence length ξ



Coherence length is the distance between the carriers creating a Cooper-Pair.

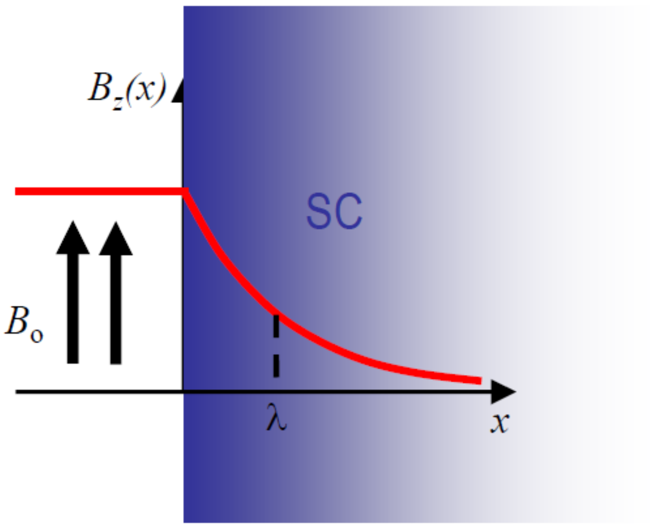
	T_c	λ [nm]	ξ[nm]	κ
Al	1.2	16	1600	0.01
Sn	3.7	34	230	0.16
Pb	7.2	37	83	0.4

	T_c	λ [nm]	ξ[nm]	κ
Nb	9.3	39	38	1
Nb_3Sn	18	80	3	27
$YBa_2Cu_3O_7$	93	150	1.5	100
Rb_3C_{60}	30	247	2.0	124
$Bi_2Sr_2Ca_2Cu_3O_{10}$	110	200	1.4	143

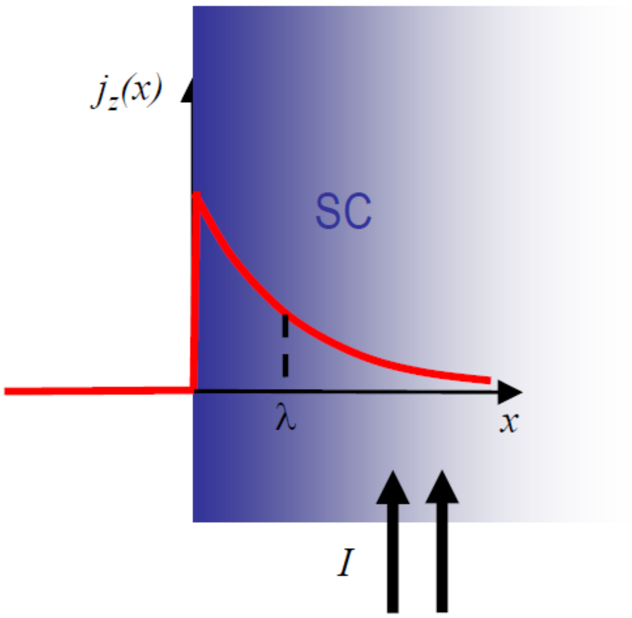
Important length scales in a superconductor

Magnetic penetration depth λ

$$\Delta \mathbf{B} = \frac{1}{\lambda^2}\mathbf{B}$$



$$\Delta \mathbf{j} = \frac{1}{\lambda^2}\mathbf{j}$$



$$\boxed{B_z(x) = B(0)\exp(-x/\lambda)}$$

$$\boxed{j_z(x) = \frac{I}{2\pi R\lambda}\exp(-x/\lambda)}$$

$$\boxed{\lambda = \sqrt{\frac{m^*}{\mu_0 e^2 n_s}}}$$

The solution of the London Eq. (2.12a) can be written as

$$B(x) = B_1 \exp\left(-\frac{x}{\lambda_L}\right) + B_2 \exp\left(+\frac{x}{\lambda_L}\right)$$

The boundary conditions are

$$B\left(-\frac{d}{2}\right) = B_0 : \quad B_1 \exp\left(\frac{d}{2\lambda_L}\right) + B_2 \exp\left(-\frac{d}{2\lambda_L}\right) = B_0$$

$$B\left(+\frac{d}{2}\right) = B_0 : \quad B_1 \exp\left(-\frac{d}{2\lambda_L}\right) + B_2 \exp\left(+\frac{d}{2\lambda_L}\right) = B_0$$

From here we calculate

$$B_1 = B_2 = \frac{B_0}{\cosh\left(\frac{d}{2\lambda_L}\right)}$$

Thus, the final expression for the field $B(x)$ inside the superconducting film is

$$B(x) = B_0 \frac{\cosh\left(\frac{x}{\lambda_L}\right)}{\cosh\left(\frac{d}{2\lambda_L}\right)}.$$